Data analysis

<u>Question 1</u>.

In Figure 1, part of the constellation of Ursa Major is shown. It was taken with a digital camera with a large CCD chip (17mm × 22mm). Find the focal length, f, of the optical system and give the error of your results.

Figure 1. Part of the constellation of Ursa Major.

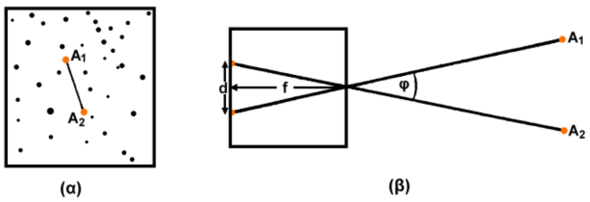


Figure 1A.Calculating the distance scale.

Answer: Calculate the angular distance, φ, (in degrees or minutes of arc), of two bright stars, whose coordinates are given in the *List of Bright stars*. Preferably these stars should be chosen to be far apart (e.g. α *UMa* [$a_1 = 11^h03^m$, $\delta_1 = +61°45'$] and η *UMa* [$a_2 = 13^h48^m$, $\delta_2 = +49°19'$]).

1. In order to calculate the angular distance of the two stars, the coordinates should be converted to decimal degrees (e.g. α *UMa* [$a_1 = 165°.75$, $\delta_1 = +61.75°$] and η *UMa* [$a_2 = 207.0°$, $\delta_2 = +49°.32$]) (**2.5 Point**)

2. Use the *cosine law* to calculate the angular distance between the two stars:

$\varphi = \arccos(\sin\delta_1 \times \sin\delta_2 + \cos\delta_1 \times \cos\delta_2 \times \cos(a_1 - a_2)$ → $\varphi = 25°.8448$ (**10 Points**)

3. Measure the distance, d_0, of the two stars in mm in Figure 1. $d_0 = 138$ mm. (**2.5 Points**

4. The photograph in Figure 1 does not have the same dimensions as the original photograph. Measure the length, ℓ, of the photograph in mm. $\ell = 140$ mm. This length corresponds to 22 mm. Convert d_0 to the distance, d, of the original photograph, $d = 138 \times \frac{22}{140}$ mm. $d = 21.6857$ mm (**1.5 Points**)

4. The *image scale* $\left(\frac{\varphi}{d}\right)$ of the original photograph is $\frac{\varphi}{d} = \frac{25^o.8448}{21.6857} = 1.1918 \frac{deg}{mm}$ (**2.5 Points**)

5. The focal length is given from equation (see Figure 2): $\tan\frac{\varphi}{2} = \frac{d}{2f}$ or $f = \frac{d}{2\tan\frac{\varphi}{2}}$ from

which the focal length is calculated: $f = \frac{21.6857}{2\times 0.2294}$ or $\boldsymbol{f = 47.3}$ **mm** (**1.5 Points**)

(If errors are included **(4 Points)**

Question 2.

You are given 5 recent photographs of the solar photosphere shot at exactly the same time every two days (May 1 – May 9, 2013) in equatorial coordinates. You are also given two transparent Stonyhurst grids, which display heliocentric coordinates (heliocentric longitude, $\ell_\odot$, and heliocentric latitude, $b_\odot$). They cover the interval between April 28 to May 15. As the Earth does not orbit exactly around the Sun's equator, so, through the year, the solar equator seems to move up and down a little more than 7 degrees from the centre of the solar disc. This angle, B_0, varies sinusoidally through the year. Furthermore, the axis of rotation of the Sun, as seen from the Earth, does not coincide with the axis of rotation of the Earth. The angle on the plane of the sky between the two axes, P_0, also varies though the year. The numerical value of these angles (B_0 and P_0) are indicated on each of the 5 image of the Sun.

(1) Mark the axis of rotation of the Sun on each photograph.

(2) Choose 3 prominent sunspots that can be followed in all (or most) photographs and mark them as *S1*, *S2* and *S3* on the photos. Using the appropriate Stonyhurst grids, find their coordinates ($\ell_\odot, b_\odot$) for every day (May 1 to May 9) and note them down in Table 1.

Table 1

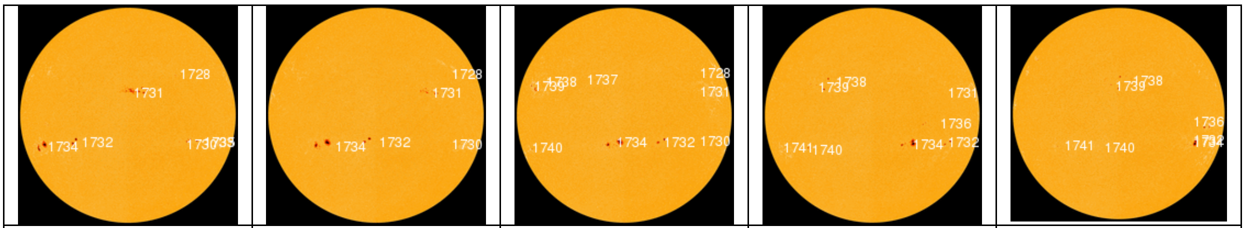
Date	Sunspot S1		Sunspot S2		Sunspot S3	
	$\ell_\odot$	$b_\odot$	$\ell_\odot$	$b_\odot$	$\ell_\odot$	$b_\odot$
May 1						
May 3						
May 5						
May 7						
May 9						

(3) Construct the diagrams $\Delta\ell_\odot/\Delta t$ for each sunspot.

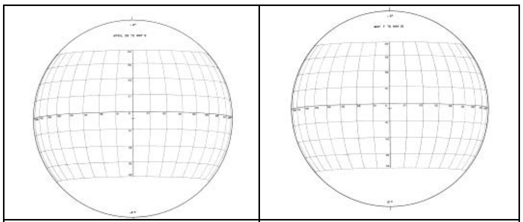
(4) Calculate its synodic period (P) of rotation **in days** for each sunspot. Write down the result for each sunspot, P_{S1}, P_{S2}, P_{S3}.

(5) Calculate the average synodic period ($P_\odot$) of rotation of the Sun in days.

Answer: Five large photographs of the solar photosphere (adapted from www.spaceweather.com) and the appropriate (and on-scale) Stonyhurst grids will be given to the students (Figure 3A)**.** The photographs given to the students, will not be annotated. Each photograph is accompanied by the angle B_0 and P_0 of the day of the observation. The student should calculate the rotation of the Sun by measuring the coordinates of at least 3 well recognized sunspots as they follow the rotation of the Sun.



1 May 2013 | 3 May 2013 | 5 May 2013 | 7 May 2013 | 9 May 2013



April 28 to May 06 | May 07 to May 15

Figure 3A. Five photographs of the solar photosphere (top) and the corresponding Stonyhurst grids

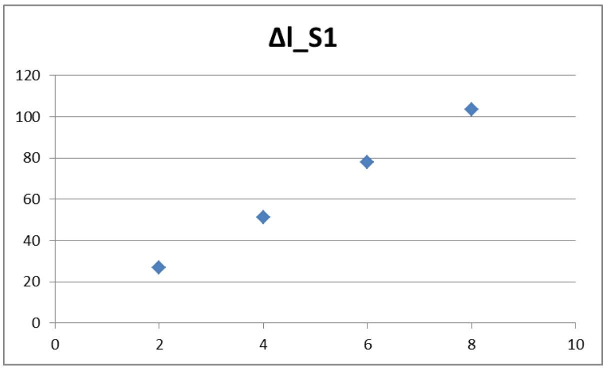
1. Correctly draw the axis of rotation of the Sun. This can be done by drawing a straight line at an angle of P_0 degrees anticlockwise from the vertical for each photograph (note: the photographs are given in equatorial coordinates.) **(4 Points)**

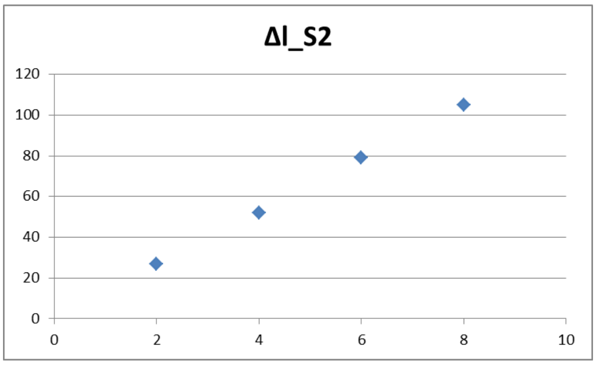
2. Choose the correct Stonyhurst grid and place it on each photograph, so that the solar axis on the grid coincides with the solar axis of the photograph. Estimate the heliocentric longitude, ($\ell_\odot$), and heliocentric latitude ($b_\odot$) with the help of the grid. Write down these coordinates in Table 1 for each of the 5 photographs.

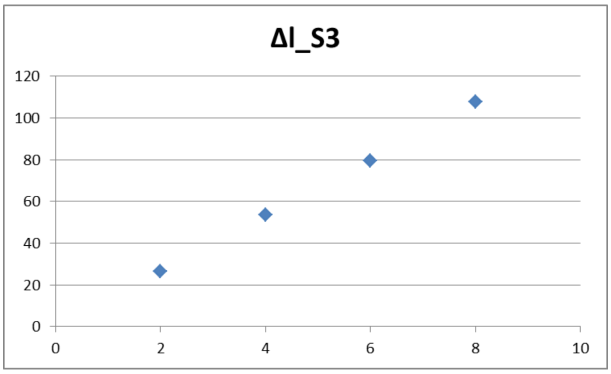
Date	Sunspot S1		Sunspot S2		Sunspot S3	
	$\ell_\odot$	$b_\odot$	$\ell_\odot$	$b_\odot$	$\ell_\odot$	$b_\odot$
May 1	54	17.5	33	18.5	30.5	16
May 3	27	18	6	19	4	16
May 5	3	18.5	-19	18.5	-23	17
May 7	-24	18	-46	18	-49	16
May 9	-49.5	19	-72	18	-77	17

(12 Points)

3. Construct the diagram $\Delta\ell_{\odot}/\Delta t$ for each sunspot:







(2 for each graph) **(6 Points)**

4. Calculate the rotation ($\Delta\ell_{\odot}/\Delta t$) of the Sun for each sunspot.

$$P_i = 8 * \frac{360}{\Delta\ell_{\odot i}}, \qquad P_1 = 27.8\ days, \qquad P_2 = 27.4\ days, \qquad P_3 = 26.8\ days$$

(3 for each graph) **(9 Points)**

5. Calculate the average period of the Sun

$P_{\odot} = (P_1 + P_2 + P_3)/3 = 27.3\ days$ **27.3 days** **(1.5 Point)**

Question 3.

Figure 2 shows a photograph of the sky in the vicinity of the Hyades open cluster. The V-filter in the Johnson's photometric system was used. Figure 3 is a chart of the region with known V-magnitudes (m_V) of several stars (note that in order to avoid confusion with the stars, no decimal point is used, i.e. a magnitude $m_V = 8.1$ is noted as "81"). Hint: some of the stars may not be in the chart.
(a) Identify as many of the *stars shown with a number and an arrow* in Figure 3 and mark them on Figure 2.
(b) Comparing the V-magnitudes of the known stars in Figure 2, estimate the V-magnitudes of the *stars shown with a number and an arrow* in Figure 3.

Answer: The human eye can easily recognize differences of the order of $\Delta m_V = 0.1 – 0.2$ mag. The student should first align the photograph with the chart, which has a different scale and orientation. Then the magnitudes of the stars can easily be recognized. Note: not all arrowed stars can be found in the photograph. (0.5 Point for each star that has been correctly identified and 0.5 point for every correct magnitude within ±0.3 magnitude **(18 Points)**

Star	3	4	5	6	7	8	9	10	11
m	7.3 – 7.9	5.3 – 5.9	5.2 – 5.8	4.6 – 5.2	5.2 – 5.8	4.5 – 5.1	4.3 – 4.9	4.9 5.5	5.3 – 5.9

Stars 1, 2 and 12 are outside the boundaries of the photograph

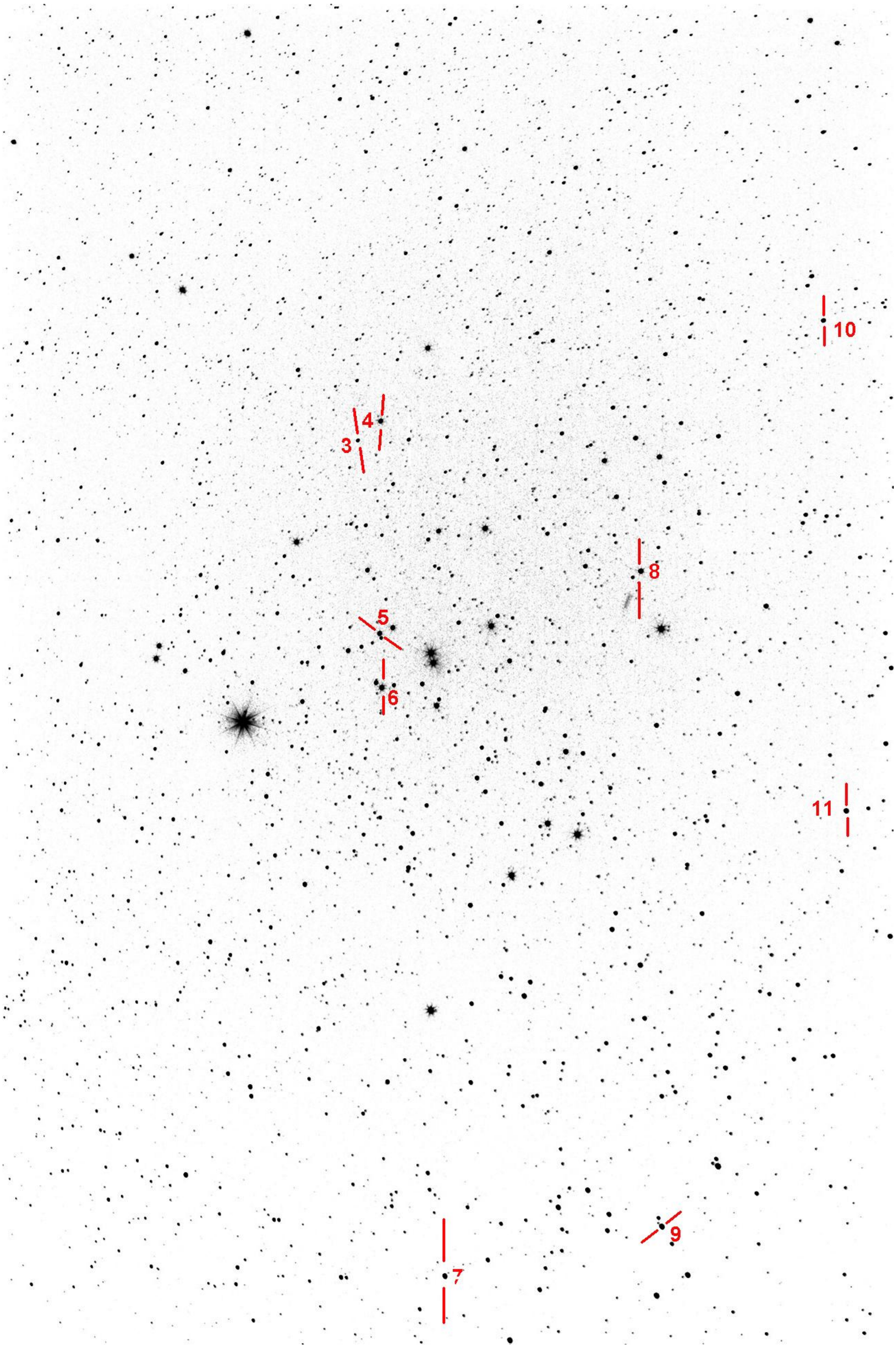


Figure 2. Photograph of the sky in the vicinity of the Hyades open cluster

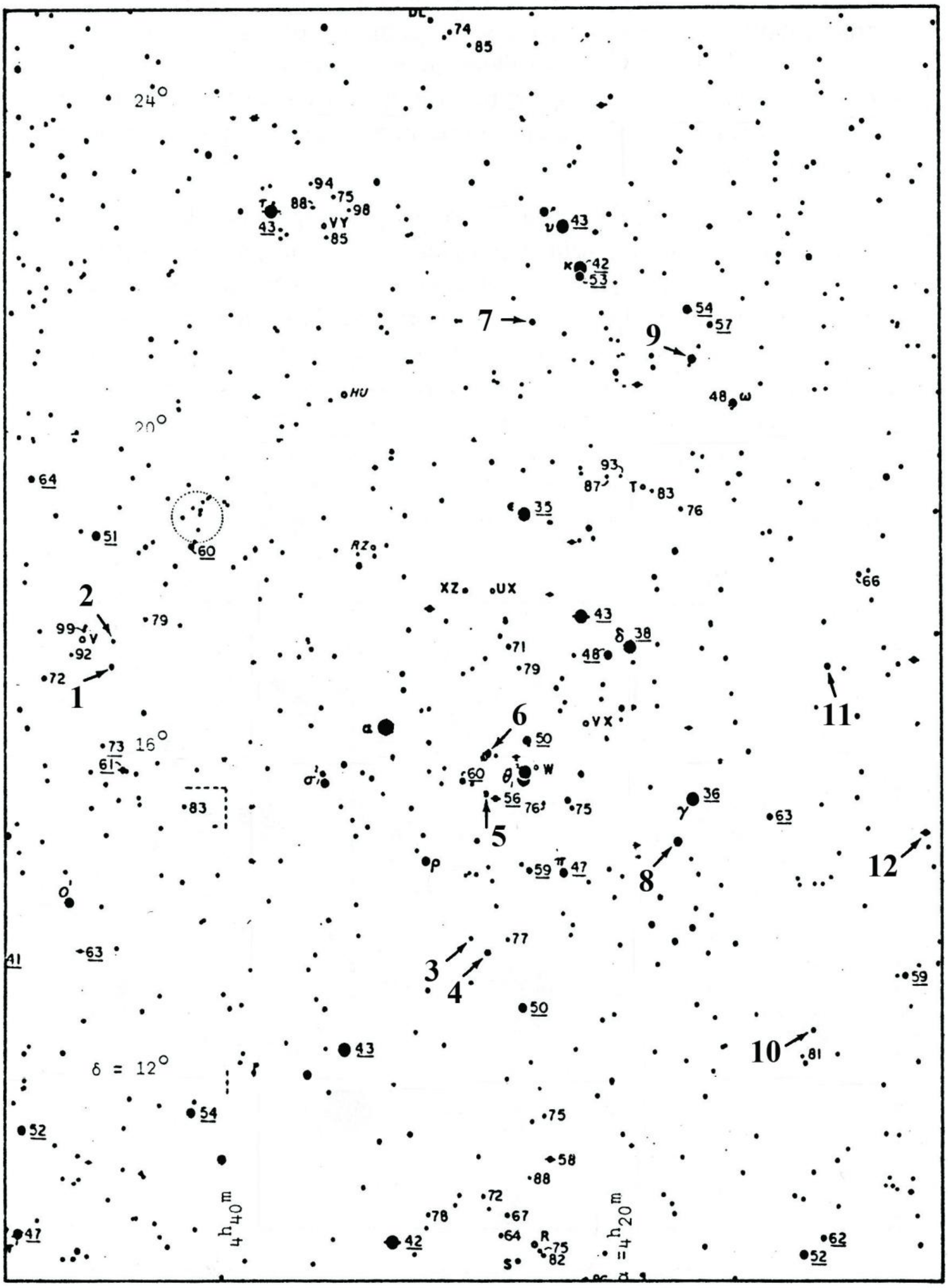


Figure 3. Sky chart of the region in the vicinity of the Hyades open cluster

Question 4.

Calculate the distance of the Hyades cluster using the *moving cluster* method (Figure 5).

1. In a *Text* file (*Hyades-stars.txt*) you are given a list of 35 stars from the field of the Hyades open cluster, observed by the Hipparcos space telescope.

The information listed in the columns of the text file for each of the 35 stars is: (a) The Hipparcos catalogue number (*HIP*). (b) Their *right ascension* (alpha – α) [h m s]. (c) Their *declination* (delta – d) [° ′ ″]. (d) Their *trigonometric parallax* (p – π) [″ × 10^3]. (e) Their *proper motion in right ascension* multiplied by cos d (mu_axcosd – $\mu_\alpha \times \cos d$) [″ × 10^3/yr]. (f) Their *proper motion in declination* (mu_d – μ_d) [″ × 10^3/yr]. (g) Their radial velocity (v_r – v_r) [km/s].

HIP	alpha	delta	p	mu_axcosd	mu_d	v_r
13834	2 58 5.08	20 40 7.7	31.41	234.79	-31.64	28.10
14838	3 11 37.67	19 43 36.1	19.44	154.61	-8.39	24.70
18170	3 53 9.96	17 19 37.8	24.14	143.97	-29.93	35.00
18735	4 0 48.69	18 11 38.6	21.99	129.49	-28.27	31.70
19554	4 11 20.20	5 31 22.9	25.89	146.86	5.00	36.60
20205	4 19 47.53	15 37 39.7	21.17	115.29	-23.86	39.28
20261	4 20 36.24	15 5 43.8	21.20	108.79	-20.67	36.20
20400	4 22 3.45	14 4 38.1	21.87	114.04	-21.40	37.80
20455	4 22 56.03	17 32 33.3	21.29	107.75	-28.84	39.65
20542	4 24 5.69	17 26 39.2	22.36	109.99	-33.47	39.20
20635	4 25 22.10	22 17 38.3	21.27	105.49	-44.14	38.60
20711	4 26 18.39	22 48 49.3	21.07	108.66	-45.83	35.60
20713	4 26 20.67	15 37 6.0	20.86	114.66	-33.30	40.80
20842	4 28 0.72	21 37 12.0	20.85	98.82	-40.59	37.50
20885	4 28 34.43	15 57 44.0	20.66	104.76	-15.01	40.17
20889	4 28 36.93	19 10 49.9	21.04	107.23	-36.77	39.37
20894	4 28 39.67	15 52 15.4	21.89	108.66	-26.39	38.90
20901	4 28 50.10	13 2 51.5	20.33	105.17	-15.08	39.90
21029	4 30 33.57	16 11 38.7	22.54	104.98	-25.14	41.00
21036	4 30 37.30	13 43 28.0	21.84	108.06	-19.71	38.80
21039	4 30 38.83	15 41 31.0	22.55	104.17	-24.29	39.56
21137	4 31 51.69	15 51 5.9	22.25	107.59	-32.38	36.00
21152	4 32 4.74	5 24 36.1	23.13	114.15	6.17	39.80
21459	4 36 29.07	23 20 27.5	22.60	109.97	-53.86	43.30
21589	4 38 9.40	12 30 39.1	21.79	101.73	-14.90	44.70
21683	4 39 16.45	15 55 4.9	20.51	82.40	-19.53	35.60
22044	4 44 25.77	11 8 46.2	20.73	98.87	-13.47	39.60
22157	4 46 1.70	11 42 20.2	12.24	67.48	-7.09	43.00
22176	4 46 16.78	18 44 5.5	10.81	73.03	-69.79	44.11
22203	4 46 30.33	15 28 19.6	19.42	91.37	-24.72	42.42
22565	4 51 22.41	18 50 23.8	17.27	79.66	-32.76	36.80
22850	4 54 58.32	19 29 7.6	14.67	63.32	-28.41	38.40
23497	5 3 5.70	21 35 24.2	20.01	68.94	-40.85	38.00
23983	5 9 19.60	9 49 46.6	18.54	63.54	-7.87	44.16
24019	5 9 45.06	28 1 50.2	18.28	55.86	-60.57	44.90

Import the *txt* file in *MS Excel*

2. Convert the coordinates in degrees (with 4 decimal points).

3. Calculate the angular distance, φ, between each of the stars and the point of convergence, which is at ($\alpha_c = 6^h7^m$, $\delta_c = +6°56'$).

4. Calculate the proper motion of each star, μ [″/yr], using $\mu_\alpha\cos\delta$ and μ_δ given in the list.

5. Use the above data to calculate the distance, r_μ, for each star using the following equation:

$$r_\mu = \frac{v_r \tan\varphi}{4.74047\mu}$$

where r_μ is the distance of the star in parsecs, v_r is the radial velocity of the star in km/sec, φ is the *angular distance* between the star and the point of convergence that you have already estimated in step 3, while μ is the total proper motion estimated in step 4. Do all stars belong to the Hyades cluster? You can assume that any stars whose distance from the centre of the cluster (r_μ = 46.34pc) is larger than 10 pc, are not part of the cluster.

6. Independently, calculate the distance, r_π, of each star in the list using the trigonometric parallax angle, π.

7. Find the average distance of the Hyades cluster, $\overline{r_\mu}$ and $\overline{r_\pi}$, and its standard deviation, σ_μ and σ_π, for each method (*moving cluster* and *trigonometric parallax* methods).

8. Which method is more accurate: (*i*) the *moving cluster* method, (*ii*) the *trigonometric parallax* method? Please answer with (*i*) or (*ii*).

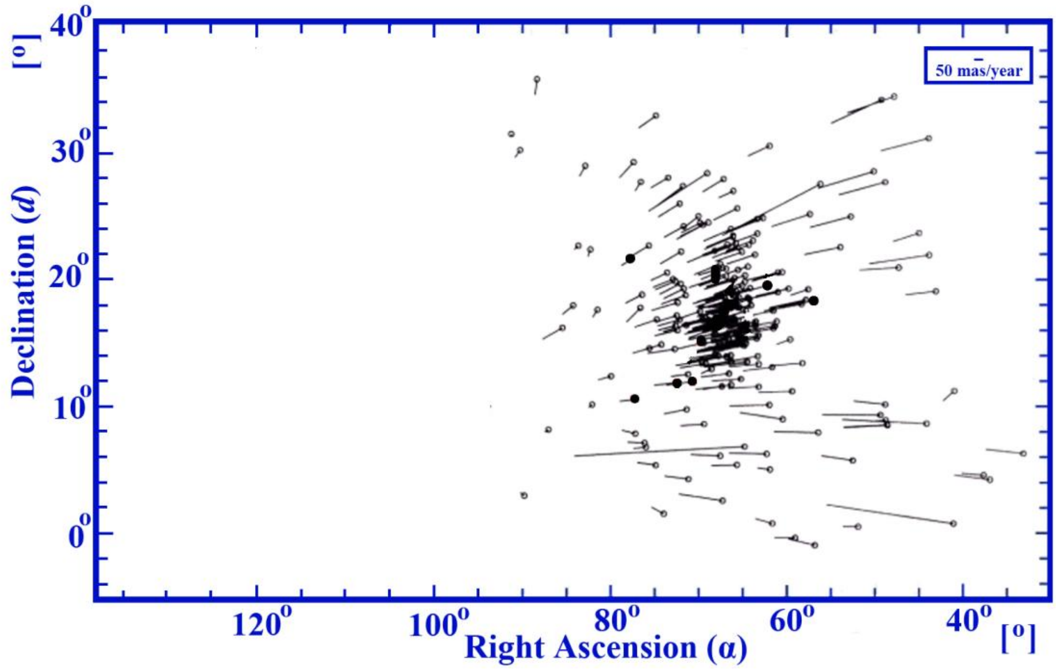


Figure 4. The distance of the Hyades open cluster using the *moving cluster* method

Answer: (1) The student should be able to import the ascii data of the *txt* file in the *MS Excel* spreadsheet application. **(12 Points)**

(2) In order to calculate the angular distance φ (next step), all coordinates should be converted into decimal degrees. The students should be able to convert the [hours, min, sec] and the [°, ′, ″] into decimal degrees, using . For the right ascension: $\alpha_{\deg} = h \times 15 + \frac{m}{60} + \frac{s}{3600}$ and for the declination: $\delta_{\deg} = {}^{\circ} + \frac{'}{60} + \frac{''}{3600}$. All coordinates are positive, so the student does not have to worry about checking the sign, which is not trivial. **(12 Points)**

(3) Using the cosine law, $\varphi = \arccos(\sin\delta_1 \times \sin\delta_2 + \cos\delta_1 \times \cos\delta_2 \times \cos(a_1 - a_2)$, the angular distance, φ, between each of the stars and the point of convergence should be calculated **(15 Points)**

(4) The student should easily calculate the proper motion of each star, by inserting the given equation, $\mu = \sqrt{(\mu_a \cos\delta)^2 + \mu_d^2}$, in the spreadsheet. **(9 Points)**

(5) Again, inserting the given equation, $r_\mu = \frac{v_r \tan\varphi}{4.74047\mu}$ in the spreadsheet (remembering to divide the given values of μ by 1000 to get arcseconds), the student should be able to calculate the distance, r_μ, of each star. Any star whose distance from the centre of the cluster is larger than 10 pc should be omitted from the following calculations. **(12 Points)**

(6) The trigonometric parallax distance is given by $r_\pi = \frac{1}{\pi \times 10^{-3}}$. This equation should be inserted in the spreadsheet. The result is in parsecs [pc]. **(6 Points)**

(7) Using the statistical functions *AVERAGE* and *STDEV* of MS Excel, the student should easily calculate the *average* and the *standard deviation* of r_{μ} and r_{π}. **(6 Points)**

(8) The method with the smaller standard deviation is, obviously more accurate. **(3 Point)**

(*macros* are allowed)

Point of convergence		R.A.	6	7	90.1167		φ = arccos(sin d_1×sin d
		Dec.	6	56	6.9333		

HIP	alpha			delta			parallax	mu_axcosc
	h	m	s	deg	arcmin	arcsec		marcsec
13834	2	58	5.08	20	40	7.7	31.41	234.79
14838	3	11	37.67	19	43	36.1	19.44	154.61
18170	3	53	9.96	17	19	37.8	24.14	143.97
18735	4	0	48.69	18	11	38.6	21.99	129.49
19554	4	11	20.2	5	31	22.9	25.89	146.86
20205	4	19	47.53	15	37	39.7	21.17	115.29
20261	4	20	36.24	15	5	43.8	21.2	108.79
20400	4	22	3.45	14	4	38.1	21.87	114.04
20455	4	22	56.03	17	32	33.3	21.29	107.75
20542	4	24	5.69	17	26	39.2	22.36	109.99
20635	4	25	22.1	22	17	38.3	21.27	105.49
20711	4	26	18.39	22	48	49.3	21.07	108.66
20713	4	26	20.67	15	37	6	20.86	114.66
20842	4	28	0.72	21	37	12	20.85	98.82
20885	4	28	34.43	15	57	44	20.66	104.76
20889	4	28	36.93	19	10	49.9	21.04	107.23
20894	4	28	39.67	15	52	15.4	21.89	108.66
20901	4	28	50.1	13	2	51.5	20.33	105.17
21029	4	30	33.57	16	11	38.7	22.54	104.98
21036	4	30	37.3	13	43	28	21.84	108.06
21039	4	30	38.83	15	41	31	22.55	104.17
21137	4	31	51.69	15	51	5.9	22.25	107.59
21152	4	32	4.74	5	24	36.1	23.13	114.15
21459	4	36	29.07	23	20	27.5	22.6	109.97
21589	4	38	9.4	12	30	39.1	21.79	101.73
21683	4	39	16.45	15	55	4.9	20.51	82.4
22044	4	44	25.77	11	8	46.2	20.73	98.87
22157	4	46	1.7	11	42	20.2	12.24	67.48
22176	4	46	16.78	18	44	5.5	10.81	73.03
22203	4	46	30.33	15	28	19.6	19.42	91.37
22565	4	51	22.41	18	50	23.8	17.27	79.66
22850	4	54	58.32	19	29	7.6	14.67	63.32
23497	5	3	5.7	21	35	24.2	20.01	68.94
23983	5	9	19.6	9	49	46.6	18.54	63.54
24019	5	9	45.06	28	1	50.2	18.28	55.86

$'_2 + \cos d_1 \times \cos d_2 \times \cos (a_1 - a_2)$ r0 46.3400

mu_d marcsec	v_r	alpha [deg	dec [deg]	phi [deg]	mu marcsec	r(mu)	r(p)	r(p)-r(m)	r(mu)-r0
-31.64	28.1	30.9681	20.6688	58.7411	236.9123	41.2182	31.8370	-9.3812	-5.1218
-8.39	24.7	45.1938	19.7267	45.3821	154.8375	34.1029	51.4403	17.3374	-12.2371
-29.93	35	45.8861	17.3272	44.3591	147.0482	49.0988	41.4250	-7.6737	2.7588
-28.27	31.7	60.0135	18.1941	31.3998	132.5400	30.7966	45.4752	14.6786	-15.5434
5	36.6	60.1889	5.5230	29.7796	146.9451	30.0661	38.6250	8.5588	-16.2739
-23.86	39.28	60.3299	15.6277	30.4351	117.7331	41.3499	47.2367	5.8867	-4.9901
-20.67	36.2	60.3434	15.0955	30.3047	110.7362	40.3045	47.1698	6.8653	-6.0355
-21.4	37.8	60.3676	14.0773	30.0788	116.0305	39.8030	45.7247	5.9217	-6.5370
-28.84	39.65	60.3822	17.5426	30.8759	111.5429	44.8354	46.9704	2.1350	-1.5046
-33.47	39.2	60.4016	17.4442	30.8306	114.9697	42.9281	44.7227	1.7946	-3.4119
-44.14	38.6	60.4228	22.2940	32.4706	114.3524	45.3124	47.0146	1.7022	-1.0276
-45.83	35.6	60.4384	22.8137	32.6692	117.9296	40.8337	47.4608	6.6271	-5.5063
-33.3	40.8	60.4391	15.6183	30.3306	119.3977	42.1745	47.9386	5.7641	-4.1655
-40.59	37.5	60.4669	21.6200	32.1677	106.8314	46.5719	47.9616	1.3897	0.2319
-15.01	40.17	60.4762	15.9622	30.3765	105.8299	46.9330	48.4027	1.4697	0.5930
-36.77	39.37	60.4769	19.1805	31.2887	113.3592	44.5251	47.5285	3.0034	-1.8149
-26.39	38.9	60.4777	15.8709	30.3534	111.8187	42.9752	45.6830	2.7078	-3.3648
-15.08	39.9	60.4806	13.0476	29.7955	106.2456	45.3621	49.1884	3.8263	-0.9779
-25.14	41	60.5093	16.1941	30.4021	107.9482	47.0107	44.3656	-2.6451	0.6707
-19.71	38.8	60.5104	13.7244	29.8790	109.8428	42.8111	45.7875	2.9764	-3.5289
-24.29	39.56	60.5108	15.6919	30.2804	106.9644	45.5544	44.3459	-1.2085	-0.7856
-32.38	36	60.5310	15.8516	30.2990	112.3569	39.4947	44.9438	5.4491	-6.8453
6.17	39.8	60.5347	5.4100	29.4452	114.3166	41.4596	43.2339	1.7743	-4.8804
-53.86	43.3	60.6081	23.3410	32.7494	122.4512	47.9793	44.2478	-3.7315	1.6393
-14.9	44.7	60.6359	12.5109	29.5662	102.8154	52.0285	45.8926	-6.1358	5.6885
-19.53	35.6	60.6546	15.9180	30.1995	84.6828	51.6128	48.7567	-2.8561	5.2728
-13.47	39.6	60.7405	11.1462	29.3006	99.7834	46.9812	48.2393	1.2580	0.6412
-7.09	43	60.7671	11.7056	29.3353	67.8514	75.1298	81.6993	6.5696	28.7898
-69.79	44.11	60.7713	18.7349	30.8814	101.0150	55.0891	92.5069	37.4178	8.7491
-24.72	42.42	60.7751	15.4721	29.9820	94.6549	54.5418	51.4933	-3.0485	8.2018
-32.76	36.8	60.8562	18.8399	30.8390	86.1332	53.8098	57.9039	4.0941	7.4698
-28.41	38.4	60.9162	19.4854	30.9995	69.4014	70.1306	68.1663	-1.9643	23.7906
-40.85	38	75.0516	21.5901	20.6527	80.1339	37.7052	49.9750	12.2698	-8.6348
-7.87	44.16	75.1554	9.8296	15.0796	64.0255	39.2026	53.9374	14.7349	-7.1374
-60.57	44.9	75.1625	28.0306	25.4054	82.3958	54.5970	54.7046	0.1076	8.2570

average	45.8380	49.7716
stdev	9.0740	11.0183

without distant stars

r(p)-r0	r(m)-<r(mu	r(p)-<r(p)>	using r0 r(mu)	using r0 r(p)	using <r(m r(mu)	using <r(p)> r(p)
-14.5030	-4.6198	-17.9346	41.2182	31.8370	41.2182	31.8370
5.1003	-11.7351	1.6688	34.1029	51.4403	34.1029	51.4403
-4.9150	3.2608	-8.3466	49.0988	41.4250	49.0988	41.4250
-0.8648	-15.0414	-4.2964	30.7966	45.4752	30.7966	45.4752
-7.7150	-15.7719	-11.1466	30.0661	38.6250	30.0661	38.6250
0.8967	-4.4881	-2.5349	41.3499	47.2367	41.3499	47.2367
0.8298	-5.5335	-2.6018	40.3045	47.1698	40.3045	47.1698
-0.6153	-6.0350	-4.0468	39.8030	45.7247	39.8030	45.7247
0.6304	-1.0026	-2.8012	44.8354	46.9704	44.8354	46.9704
-1.6173	-2.9099	-5.0489	42.9281	44.7227	42.9281	44.7227
0.6746	-0.5256	-2.7570	45.3124	47.0146	45.3124	47.0146
1.1208	-5.0043	-2.3107	40.8337	47.4608	40.8337	47.4608
1.5986	-3.6634	-1.8329	42.1745	47.9386	42.1745	47.9386
1.6216	0.7339	-1.8099	46.5719	47.9616	46.5719	47.9616
2.0627	1.0950	-1.3689	46.9330	48.4027	46.9330	48.4027
1.1885	-1.3129	-2.2431	44.5251	47.5285	44.5251	47.5285
-0.6570	-2.8628	-4.0886	42.9752	45.6830	42.9752	45.6830
2.8484	-0.4759	-0.5832	45.3621	49.1884	45.3621	49.1884
-1.9744	1.1727	-5.4060	47.0107	44.3656	47.0107	44.3656
-0.5525	-3.0269	-3.9840	42.8111	45.7875	42.8111	45.7875
-1.9941	-0.2836	-5.4257	45.5544	44.3459	45.5544	44.3459
-1.3962	-6.3433	-4.8278	39.4947	44.9438	39.4947	44.9438
-3.1061	-4.3784	-6.5377	41.4596	43.2339	41.4596	43.2339
-2.0922	2.1413	-5.5238	47.9793	44.2478	47.9793	44.2478
-0.4474	6.1905	-3.8790	52.0285	45.8926	52.0285	45.8926
2.4167	5.7748	-1.0149	51.6128	48.7567	51.6128	48.7567
1.8993	1.1432	-1.5323	46.9812	48.2393	46.9812	48.2393
35.3593	29.2918	31.9278				
46.1669	9.2511	42.7354	55.0891		55.0891	
5.1533	8.7038	1.7217	54.5418	51.4933	54.5418	51.4933
11.5639	7.9718	8.1323	53.8098		53.8098	57.9039
21.8263	24.2926	18.3948				
3.6350	-8.1327	0.2034	37.7052	49.9750	37.7052	49.9750
7.5974	-6.6354	4.1659	39.2026	53.9374	39.2026	53.9374
8.3646	8.7590	4.9330	54.5970	54.7046	54.5970	54.7046
		average	44.2142	46.5074	44.2142	46.8635
		stdev	6.2857	4.2707	6.2857	4.6593