LIGO (5 points).

The first detection of gravitational waves GW150414 was announced in 2016 by the collaboration LIGO (Laser Interferometer Gravitational-Wave Observatory). The detected signal corresponds to the merger of two black holes with masses of $35M_\odot$ and $30M_\odot$, which when joined formed a black hole of $62M_\odot$. Ignoring the rotational energies of the black holes, you may assume that the energy released by this process (E_{GW}) is emitted solely in the form of gravitational waves, that were observed by the interferometer in 2015. You are given that the explosion of a supernova (SN) releases $E_{SN} = 2 \times 10^{44}\ J$.

1.1	To find out which of these two events (SN, GW) releases more energy, estimate the energy ratio $\frac{E_{SN}}{E_{GW}}$.	5.0pt

International Olympiad on Astronomy and Astrophysics

Temperature of the Earth (10 points).

For at least the last few million years, the Earth has been in roughly thermal equilibrium with the radiation from the Sun at the Earth's orbital distance.

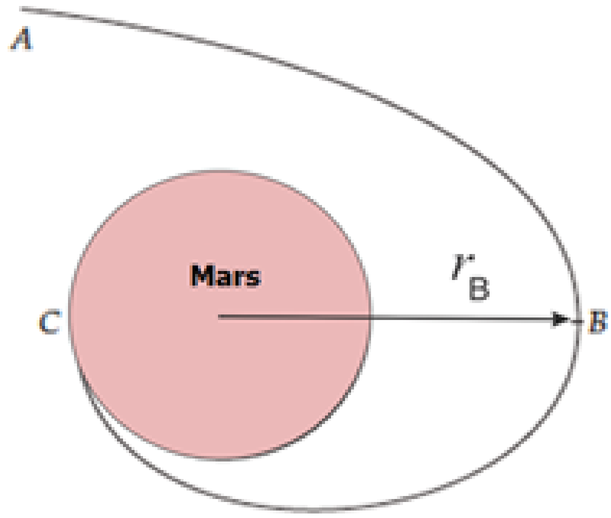
2.1	Assuming our planet to be an ideal black body, calculate what the Earth's equilibrium temperature (in Celsius) would be.	4.0pt

2.2	The Earth's albedo is approximately 30%. Calculate the Earth's surface temperature (in Celsius) considering its albedo.	2.0pt

2.3	The Earth's absorbed radiation is reemitted as black body radiation from its surface, but its atmosphere re-absorbs 58% of that energy, causing a greenhouse effect. Considering this effect, calculate the Earth's surface temperature (which will be the same as the temperature of the lower atmosphere). Give your answer in Celsius. For simplicity, consider the reabsorption effect as happening only once, and do not consider the atmosphere as a separate black body.	4.0pt

Mars (10 points).

A spacecraft of mass m=5.0×10^4 kg approaches in a parabolic orbit AB, with respect to Mars. When the spacecraft reaches point B of least distance to the center of Mars, $r_B = 6.8 \times 10^6$ m, it undergoes an instantaneous deceleration using its rockets and goes into a perfectly calculated orbit so that it will touch the Martian surface exactly at point C, diametrically opposite B, as shown in the figure.



3.1	Determine the speed ($km\ s^{-1}$) of the spacecraft at point B just before the deceleration.	3.0pt
3.2	Calculate the total energy (J) of the spacecraft as it is moving between points B and C.	4.0pt
3.3	Calculate the speed ($km\ s^{-1}$) of the spacecraft at point C.	3.0pt

ALMA - Calculating photons (10 points).

ALMA is a radio observatory with a revolutionary design. It consists of 66 high-precision antennas, operating in the wavelength range from $0.32\,mm\,to\,8.60\,mm$. The principal array has fifty antennas of $12\,m$ diameter each that can work together as a single telescope in the so-called interferometric mode. There is also another array of four $12\,m$ antennas, and twelve smaller antennas of $7\,m$ diameter each.

Imagine that a single $12\,m$ antenna is being calibrated, pointing to a source with a known incident flux of $1 \times 10^{-20}\ W/m^2$

4.1	Assuming that all the flux arrives at the shortest wavelength of ALMA sensitivity, determine the average number of photons that would reach the detector every second.	2.0pt

4.2	Compare it to the average number of photons that would have reached the detector, if all the flux arrived at the longest wavelength of operation.	2.0pt

4.3	What is the angular resolution (in arcsec) of a single $12\,m$ antenna, operating at $74.9\,GHz$?	2.0pt

4.4	Imagine the principal array operating at $74.9\,GHz$ in the interferometric mode. Assuming for simplicity that the spatial resolution is solely given by the longest baseline (largest distance between any pair of antennas), which turns to be $D_{\text{max}} = 16\,km$, what would be the angular resolution (in arcsec) in this case? Treat this case as a single slit aperture instead of a circular one.	2.0pt

4.5	For a radio antenna, the term SEFD refers to 'System Equivalent Flux Density', which is a characteristic energy flux density of the antenna, depending on its temperature and size. We also note that for energy estimation of radio photons, Rayleigh-Jeans approximation is valid. Assuming a system temperature of $691\ K$, what would be the SEFD of the full ALMA observatory in Jansky if all the 66 antennas could work together?	2.0pt

Under pressure (10 points).

Magnetic fields in the Sun are constantly shaping the structure of various different features in the Solar atmosphere. Inside any feature, the magnetic field (*B*) adds to the total pressure exerted by the gas. This so-called magnetic pressure is a function of the height *z* and can be expressed as:

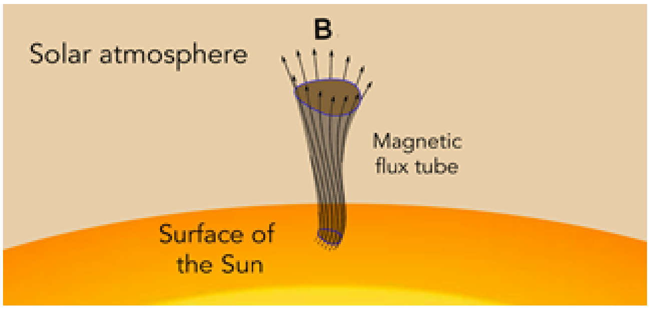
$$P_{mag}(z) = \frac{B^2(z)}{2\mu_0}$$

On the other hand, the gas can be considered to be in hydrostatic equilibrium and hence the gas pressure decays exponentially from an initial pressure value P_0 with increasing *z*. It can be expressed as,

$$P_{gas}(z) = P_0 e^{-z/H}$$

where H is the scale height, i.e. the height at which the pressure falls to $\frac{P_0}{e}$.

Consider one type of feature, a magnetic flux tube rising from the Solar surface up into an unmagnetized environment (see Figure below). Assuming that the total pressure of the material inside the tube and of the material outside it is in equilibrium,



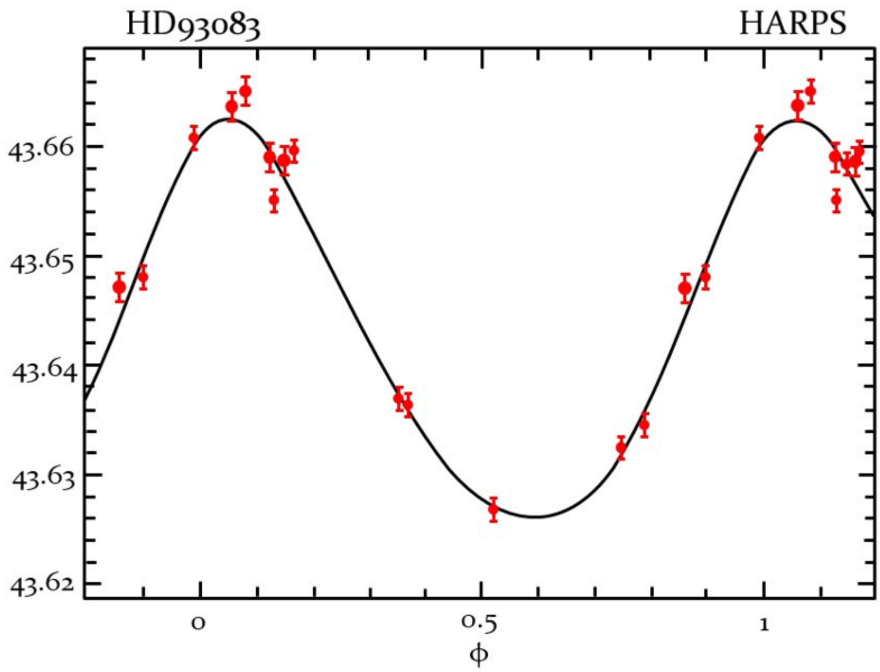
5.1	Find an expression for the magnetic field strength as a function of height z.	7.0pt

5.2	If the magnetic field at the base of a flux tube is $0.3T$, and scale height H in a given solar model is $150\ km$, at what height will the magnetic field be reduced to $0.03T$?	3.0pt

Macondo and Melquiades (12 points).

In 2019, as a part of the NameExoWorlds campaign of the International Astronomical Union, Colombia was granted an opportunity to select a name for the star HD 93083 and its planetary system. HD 93083 is a $K-type$ dwarf star and has one extrasolar planet orbiting it. Today they are officially known as Macondo (star) and Melquiades (planet), from the literary ideas of the Colombian writer Gabriel García Márquez.

This star has an effective temperature of 4995 K and an apparent visual magnitude of 8.3. As per GAIA DR2, the parallax for Macondo is 35.03 milliarcseconds. You may assume the orbit of Melquiades is perfectly circular. In the figure you can see the plot of radial velocity of Macondo with respect to the phase.



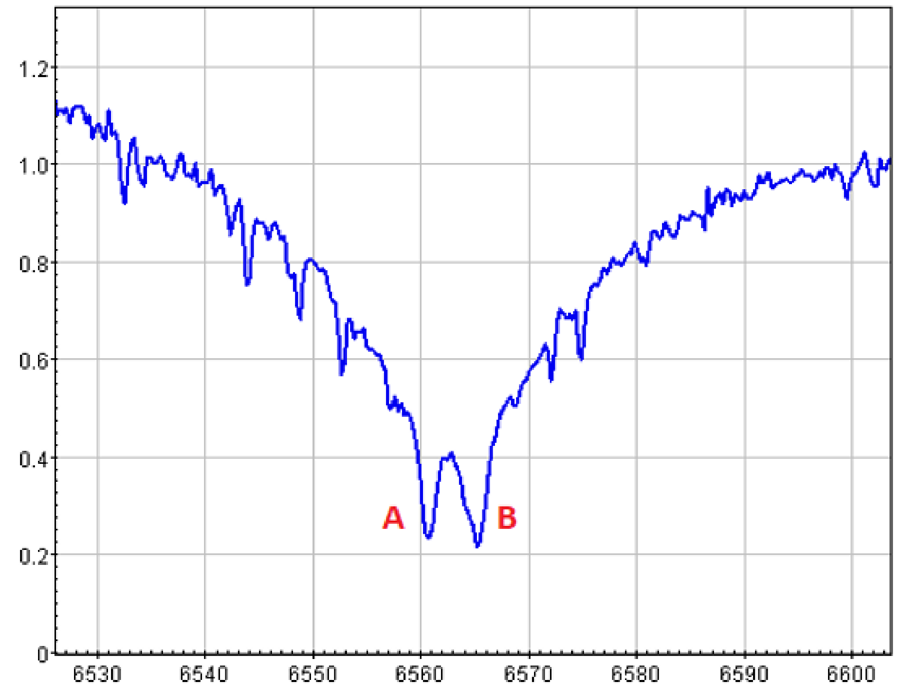
Radial velocity of Macondo (Y-axis in $km\ s^{-1}$) as a function of the phase (X-axis).

6.1	Find the wavelength $(in\,nm)$ of peak emission for Macondo in its rest frame (i.e., ignoring Doppler shifts).	2.0pt
6.2	Find the distance of this system from the Earth (in parsecs) and the absolute visual magnitude (M_V) of the star.	2.0pt
6.3	Calculate the mean radial velocity of Macondo (in $km\ s^{-1}$).	2.0pt

6.4	Calculate the orbital velocity (in km/s) of Melquiades (v_p), if mass of the star (m_s) is $0.7\ M_\odot$ and the mass of exoplanet (m_p) is $7 \times 10^{26} kg$. Assume that the orbital plane of the system is edge-on with respect to our line-of-sight.	2.0pt
6.5	Find the orbital radius of Melquiades (in au) and its orbital period (in days).	4.0pt

Menkalinan (β Aurigae) (13 points).

Almost half of the stars that we see are either binary or multiple star systems. A well-known example of this is Menkalinan (Beta Aurigae), which was initially thought to be a single star, but today recognised as a binary system comprising two stars that we will refer to as Menkalinan A and B. In the following figure, a spectrum of the system (obtained by the observatory of the Universidad de los Andes, in Bogotá) is shown:



Spectrum of Menkalinan binary system in the region of $H\alpha$. Y-axis is for the relative flux, and X-axis measures wavelengths. Menkalinan A is marked as A in the graph, and Menkalinan B is marked as B.

Answer the following questions using the plot and noting that the wavelength of $H\alpha$ line in the laboratory frame is $656.28\,nm$. Assume circular orbits, and assume that the binary system as a whole is at rest with respect to the observer.

7.1	In the spectrum, we can see the $H\alpha$ line for each star in the system. Calculate the line-of-sight velocity of each star (km/s) and determine, at the time of this observation, which of the two stars is moving towards us.	5.0pt

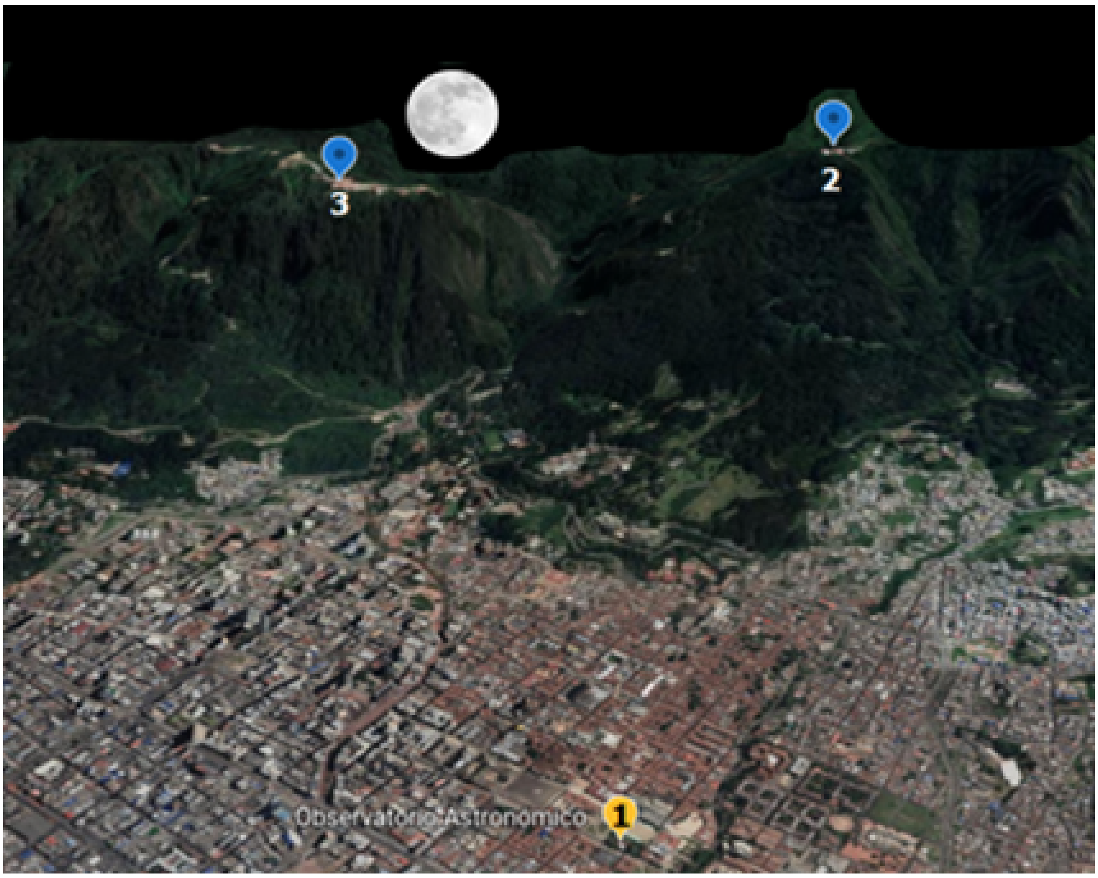
7.2	The binary system is located 81.1 light years from Earth and has an orbital period of 3.96 days. The semi-major axis for Menkalinan B (smaller star) was measured to be $3.35\,milliarcseconds$. If the mass ratio of the two components is 1.026, find the total mass of the system (in solar masses).	4.0pt

7.3	Calculate the individual masses of Menkalinan A and B in solar masses.	2.0pt

7.4	Since Menkalinan A and B are main sequence stars, use the relation: $$\frac{L}{L_{\odot}} = (\frac{M}{M_{\odot}})^{3.5}$$ to estimate the luminosity of each star (in solar luminosity).	2.0pt

IOAA Logo (15 points).

The IOAA2021 logo is formed by the acronym IOAA, where the first letter is represented by the silhouette of the building of the National Astronomical Observatory (OAN) of Colombia, the oldest observatory in America. This observatory is located in Bogota, where it was founded in 1803. The capital city of Colombia is bordered by two famous hills, Monserrate and its neighbor Guadalupe, which are icons of Bogota's cityscape that decorate the logo's background.



Aerial view of Bogota City. Numbers show locations for the quoted places: 1 is for OAN; 2 is for Guadalupe; and 3 is for Monserrate.

Point	Latitude	Longitude	Elevation (m.a.s.l)
1	$4^\circ\,35'\,53''\,N$	$74^\circ\,04'\,37''\,W$	2607
2	$4^\circ\,35'\,30''\,N$	$74^\circ\,03'\,15''\,W$	3296
3	$4^\circ\,36'\,18''\,N$	$74^\circ\,03'\,19''\,W$	3100

8.1	Estimate the distance (in km), between points 2 (Guadalupe) and 3 (Monserrate).	3.0pt

8.2	Estimate the angular separation (in degrees) between Guadalupe (2) and Monserrate (3) as observed from the National Astronomical Observatory of Colombia (1).	6.0pt

8.3	From the OAN, on September 21 at 8:00 p.m. the Moon was observed towards the eastern hills (between Monserrate and Guadalupe). The measured ecliptic coordinates (longitude and latitude) of the Moon are shown in the table. Determine the equatorial coordinates of the Moon at the time of observation.	6.0pt

Local Time: 8:00 p.m.
$Az: +90°42'59''/Alt: +19°01'42''$
$\lambda: +12°20'16''$ / $\beta: -04°24'14''$

Note: Azimuth measured from North to East.

Pluto Satellites (15 points).

9.1	The mass of Charon, the biggest satellite of Pluto, is 1/8th the mass of Pluto. Both bodies move in a circular orbit around a common center of mass. In addition, they both are tidally-locked. The distance between the center of the planet and the center of the satellite is$R = 19\,640\,km$ and radius of the satellite is $r = 593\,km$. Let g_0 be the gravitational acceleration on the surface of Charon due only to its mass.Let A be the point on Charon surface directly facing Pluto, and B the point diametrically opposite. Compute the percentage difference between gravitational acceleration at A and B respect to g_0.	15.0pt

Terrestrial Transit (15 points).

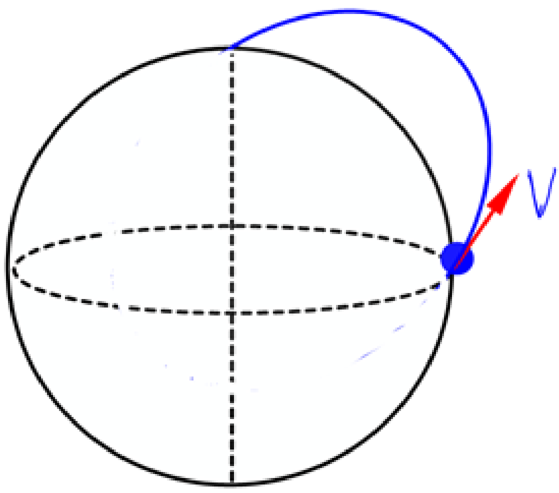
Note: Assume perfect circular orbits in both questions below.

10.1	An alien astronomer from a distant planetary system is observing the Sun. Suddenly, the brightness of the Sun drops due to the transit of the Earth in front of it. What is the maximum duration that this transit may last (in hours)? Assume that the planet where the astronomer observes from, does not move relative to the Sun.	5.0pt
10.2	Imagine that the transit of a given exoplanet as seen from Earth lasts 31 minutes. The host star is a red dwarf, with mass and radius that are 10% of the mass and radius of the Sun. What is the minimum orbital period this exoplanet may have (in days)?	10.0pt

Minimum velocity of a projectile (15 points).

11.1	What is the minimum speed with which a projectile must be launched from the Earth's surface at the equator such that the projectile reaches the north pole?	12.0pt
11.2	Find the eccentricity of the trajectory described by the projectile	3.0pt

You may ignore the rotation of the Earth. Also assume the earth surface is spherical.



Reference Chart

Hodograph (15 points).

In curvilinear motion of a planet around a star, the direction of the velocity vector changes continuously. This can be represented by a so-called "trajectory in velocity space" and is obtained as follows: for each point on the spatial trajectory, the corresponding velocity vector is drawn so that its starting point is at the origin of the velocity space, and its magnitude and direction is the same as the velocity vector at that point. The tip of this variable velocity vector generates a curve in velocity space. (The name 'hodograph' was given to this curve by Hamilton in 1846.)

As an example, see figures 1 and 2 below. For a circular orbit (Figure 1), the magnitude of the velocity is constant and therefore, the hodograph (Figure 2) of the velocity vector for Keplerian circular motion is also a circle, the center of which is located at the origin of the velocity space. The radius of this circle is equal to the constant magnitude of the circular velocity.

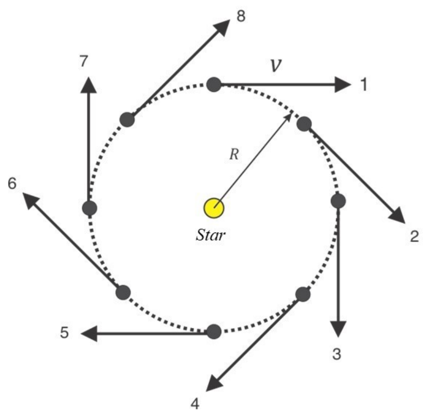


Fig. 1 Spatial trajectory of the Planet with Uniform Circular Motion around the star.

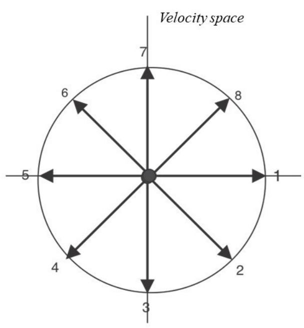


Fig. 2 Corresponding hodograph

12.1	Write an expression for the radius of the hodograph in Fig. 2, as a function of the mass M of the star, and the radius R of the circular orbit of the planet's motion.	1.0pt

12.2 For a planet in a Keplerian trajectory, write the expression for centripetal acceleration vector $(\vec{a})$ and the magnitude of angular momentum (L). For any Keplerian trajectory, it is true that **4.0pt**

$$| \Delta v | = k\Delta\theta \tag{1}$$

Where k is a constant for each type of Keplerian trajectory. Find the expression for the constant k as a function of the masses M and m of the star and the planet, respectively, and the angular momentum, L.

(Eq.1) allows us to conclude that for any Keplerian trajectory, the hodograph $(v\ as\ a\ function\ of\ \theta)$ is a circle, but except for circular motion, the centre of the hodograph does not coincide with the star. It is not necessary to prove this result, you may simply accept it as a given. For the hodograph of uniform circular motion, the compliance with (eq.1) is completely obvious, as evidenced in Fig. 3

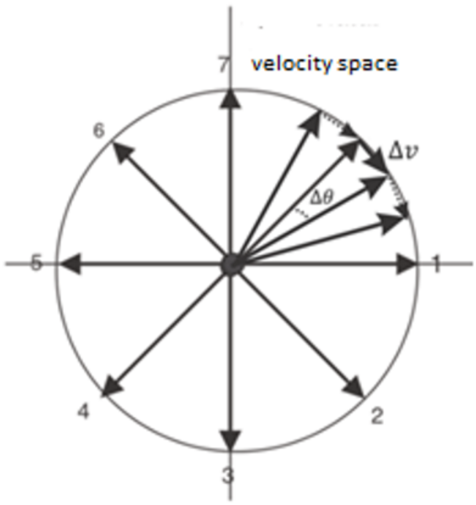


Fig. 3

12.3	Determine the expression of the constant k for the hodograph of circular planetary motion.	2.0pt

12.4 Given that the hodograph of the Keplerian elliptical motion is a circle, determine the radius of this hodograph and the distance between the center of the hodograph and the position of the star, as a function of the velocities at periastron and apoastron. Draw a rough sketch of the hodograph in the answer sheet as per the schematic shown in Fig. 4. The black circle is the star. 4.0pt

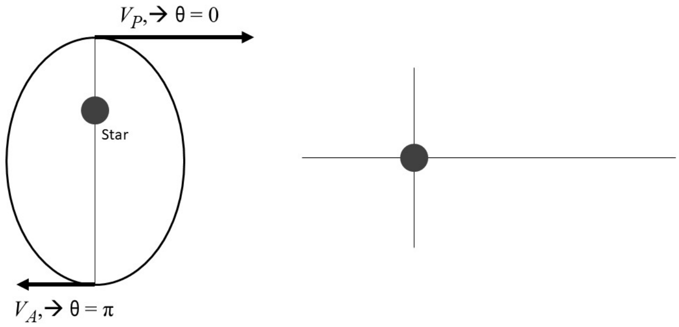


Fig. 4

12.5 Similarly, for the parabolic Keplerian trajectory, determine the radius of the corresponding hodograph and the distance from the center of that hodograph circle to the star. Express the radius as a function of the velocity at periastron. Draw a rough sketch of the hodograph circle in the answer sheet. 4.0pt

Lucy: The First Mission to the Trojan Asteroids (15 points).

CCD cameras on space probes are very sensitive and exposed to space weather conditions. Intense radiation passing through the CCD produces electron-hole pairs in the silicon of the CCD chip. The rate at which these pairs are produced is an important parameter when operating cameras on board spacecraft and can be calculated for radiation of any given energy.

A high energy particle or photon of radiation passing through the CCD will deposit some energy in the chip with each electron-hole pair it creates. The 'stopping power' of silicon for a given type of particle can be measured as the energy per areal density $(areal\ density = mass\ per\ unit\ area)$ that the silicon 'takes away' from the travelling particle.

NASA's Lucy mission will be the first to study the Trojan asteroids and will revolutionize our understanding of the formation of the Solar System. One of the instruments on board is L'LORRI (Lucy LOng Range Reconnaissance Imager), which contains a sensitive CCD in order to produce detailed images of the Trojan asteroids. Unfortunately, the radiation around Jupiter is very intense and it can generate a lot of 'noise' in the pixels of the CCD.

Let us assume that an average charged particle trapped in Jupiter's magnetic field has an energy of $15\ MeV$ and that the flux of such particles in this region is equivalent to about 600 electrons $s^{-1}\ cm^{-2}$. Also assume that for each electron-hole pair which a particle passing through a pixel creates, it deposits exactly the excitation energy of the pair in that pixel. After the pixel crosses a threshold number of electron-hole pairs it is 'excited' and no more pairs can be produced in that pixel. Any remaining energy in the particle is passed to the next pixel (and so on).

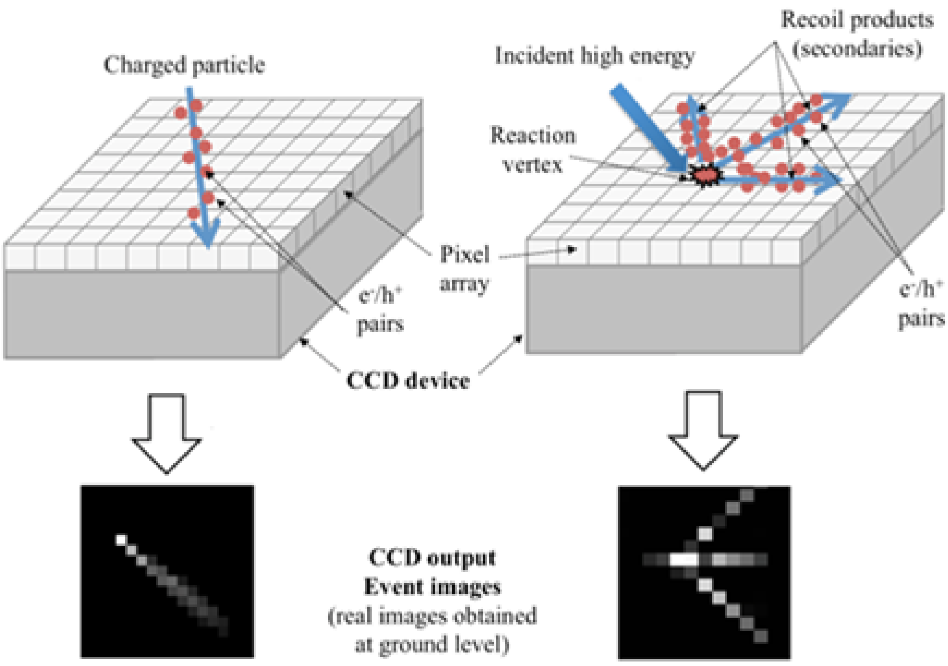
Using the data given below for the CCD chip in the L'LORRI camera, answer the following questions:

13.1	How many pixels will be excited by one such particle of radiation passing through the CCD when the spacecraft is near Jupiter's orbit?	10.0pt

13.2	Given the radiation flux near Jupiter, what percentage of the total number of pixels in an image will be excited?	5.0pt

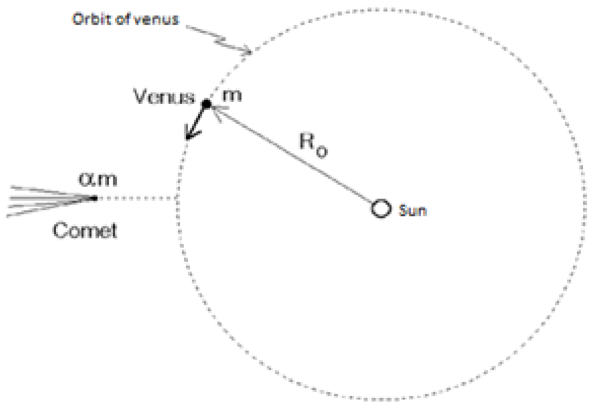
CCD Data:

- **Exposure time of an image = 30 ms**
- **Pixels on the CCD = 1024 x 1024**
- **CCD Area = 13 mm x 13 mm**
- **CCD chip thickness = 0.06 cm**
- **Density of silicon, ρ = 2.34 g cm^{-3}**
- **Excitation energy of single pair = 2.36 eV**
- **Excitation threshold of a single pixel = 250 pairs**
- **Stopping power' of silicon for a 15 MeV electron = 3.012 MeV g^{-1} cm^{2}**



Formation of the Venus-2 (35 points).

A comet of mass αm is heading ("falls") radially towards the Sun. It is known that the total mechanical energy of the comet is zero. The comet crashes into Venus, whose mass is m. We further assume that the orbit of Venus, before the collision, is circular with radius R_0. After the crash, the comet and Venus form a single object, called "Venus-2".



14.1	Find the expression in terms of M_{sun} and R_0 for the orbital speed, v_0 , of Venus before the collision.	1.0pt
14.2	Find an expression for the total mechanical energy of Venus in its orbit before colliding with the comet.	1.0pt
14.3	Find an expression for the radial velocity, v_r , the angular momentum, L , of "Venus-2" immediately after the collision.	10.0pt
14.4	Find an expression for the mechanical energy of the combined object "Venus-2" and express it in terms of energy before the collision, E_i, and α .	5.0pt
14.5	Show that the post-collision orbit of "Venus-2" is elliptical and determine the semi-major axis of the orbit.	5.0pt
14.6	Determine if the year for the inhabitants of "Venus-2" has been shortened or lengthened because of collision with the comet. Write the ratio between the period of Venus-2 and Venus.	3.0pt

14.7	What should be the value of α such that the post-collision orbit of Venus-2 would make it crash in the Sun? We will call this as α_c	5.0pt

14.8	A comet with $\alpha = \alpha_c$ collided with Venus. Calculate the percentage change in the magnitude of Venus' velocity (δv) and the change in the direction of the velocity vector $(\delta\theta)$ immediately after the collision.	5.0pt

Cosmic String (55 points).

Introduction

According to our current understanding, just after the Big Bang, when the Universe was extremely hot, the electromagnetic force, the strong nuclear force as well as the weak nuclear force were unified as one Grand Unified (GUT) force.

When the Universe cooled down to $T_{GUT} = 10^{29}\ K$, the strong nuclear force decoupled from the electroweak force. Later, when the temperature reduced to $T_{EW} = 10^{15}\ K$, the weak force decoupled from the electromagnetic force. These transitions happened in a rapid succession within a small fraction of a second after the Big Bang. It is thought that these phase transitions produced a variety of peculiar objects, called vacuum defects, which may still be observed today.

This question will discuss properties of one such possible type of defect called cosmic strings and their observational effects.

Note 1. Unless otherwise stated use the laws of Newtonian Mechanics

Note 2. You will use the following constants:

- Stefan Boltzmann Constant

$$\sigma = \frac{2\pi^5 {k_B}^4}{15\, h^3 c^2} = \frac{\pi^2 {k_B}^4}{60\, \hbar^3 c^2}$$

- The reduced Planck constant

$$\hbar = \frac{h}{2\pi}$$

- Universal Radiation Constant

$$a = \frac{4\sigma}{c} = 7.5657 \times 10^{-16} J\, m^{-3} K^{-4}$$

- Planck Temperature

$$T_{pl} = \sqrt{\frac{\hbar c^5}{G k_B^2}} = 1.416784 \times 10^{32} K$$

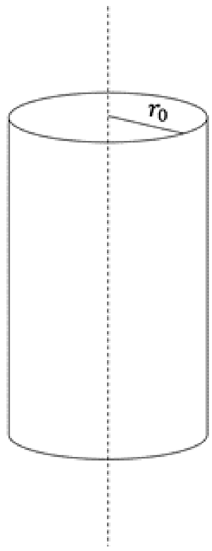
Note 3. Recall that the gravitational field $\vec{g}$ satisfies the Gauss theorem:

$$\vec{g} \cdot \vec{A} = -4\pi G M_{in}$$

Where M_{in} is the mass enclosed by the surface A.

Part A: Gravitational Field of a Cosmic String (22 points).

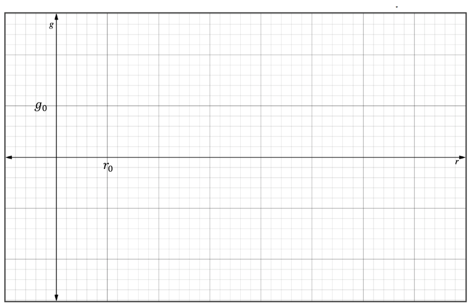
As a first approximation, let us consider a cosmic string as an infinitely long cylinder of radius r_0 and mass per unit length μ .



A.1	Write an expression in terms of the constants G, μ and $r_0$for the gravitational field produced by the string, $\vec{g}(r)$. Consider the cases $r_0 < r$ and $r_0 > r$ independently	6.0pt

A.2	Write an expression in terms of the constants G, μ and $r_0$for $g_0 \equiv \|\vec{g}(r_0)\|$.	1.0pt

A.3	Let g be defined as $\vec{g}(r) \cdot \hat{r}$. Draw a rough sketch of g vs. r in the figure given in the answer sheet	3.0pt



A.4	It is possible to define a stable orbit around a Cosmic String. For circular orbits of radius $R > r_0$ and period τ, the following relation is attained $$R = A\tau^{\alpha}$$ where A and α are constants. Find A and α in terms of G and μ		4.0pt

The following three questions refers to a classical newtonian particle moving with speed v when at a distance $r > r_0$ from the string. You will need to use the result below:

$$\int_{x_0}^{x} \frac{dx}{x} = \ln\left(\frac{x}{x_0}\right)$$

A.5	Show that the gravitational potential energy of the particle is $$U = Gm\mu \ln\left(\frac{r}{b}\right)$$ where b is any fixed distance.		3.0pt

A.6	What is the maximum distance, R_{max} , from the string, that the particle can reach?		4.0pt

A.7	Is it possible for the particle to escape the gravitational field? Write YES/NO in the answer sheet.		1.0pt

Part B: Cosmic string as a photon gas (17 points).

Consider now a cosmic string as a photon gas inside a very long cylinder of radius r_0 with adiabatic walls, and in thermal equilibrium at temperature T.

B.1	What is the energy density ρ of the string in terms of $T,\ \hbar, k_B$ and c ?	2.0pt

B.2	The radius r_0 is related to the temperature T vía $$r_0 = \frac{\hbar^{n_1}\, c^{n_2}}{k_B T},$$ where $\hbar$ is the reduced Planck constant, and c is the speed of light in vacuum, k_B is the Boltzmann constant, and n_1 and n_2 are integer numbers. Determine n_1 and n_2	4.0pt

B.3	What is the mass per unit length, μ , of the string in terms of ρ and r_0 ?	2.0pt

B.4	Express the inequality for the weak field condition, defined as $$\frac{2G\mu}{c^2} \ll 1,$$ only in terms of T and T_{pl} .	5.0pt

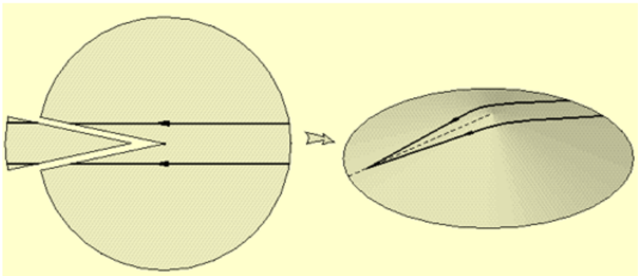
B.5	Calculate $\frac{2G\mu}{c^2}$ for $$\bullet T = T_{EW}$$ $$\bullet T = T_{GUT}$$	3.0pt

B.6	Does the weak field condition hold for T_{EW}? Answer YES or NOT. Does the weak field condition hold for T_{GUT}? Answer YES or NOT.	1.0pt

Part C: Gravitational Lensing from cosmic Strings (16 points).

So far, in part A and B , we have neglected the internal pressure of the photon gas inside the string. If we include it in our analysis, we need to consider the General Theory of Relativity.

After solving the Einstein field equations, one finds that the spacetime around a cosmic string is conical as if a narrow wedge were removed from a flat sheet and the edges connected, as shown below.



http://www.ctc.cam.ac.uk/outreach/origins/cosmic_structures_five.php

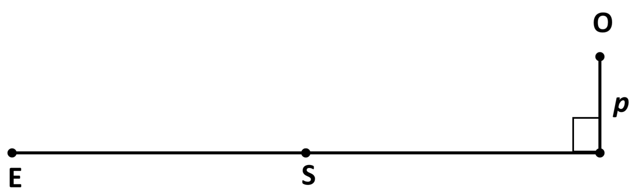
A remarkable result of this model is light deflection by a cosmic string, which leads to the possibility of detection through gravitational lensing.

The angle of deflection (in radians) of a light ray coming from a distant quasar (O in the figure below), as the light passes close to a cosmic string (S in the figure below) and eventually reaching an observer on the Earth, (E in the figure below), is

$$\delta\phi = \frac{4\pi G\mu}{c^2}$$

and is independent of the parameter, p, as shown in the figure below:

In the figure E and O are in a plane perpendicular to the string. The distance between the observer and the string is D_{ES} and the distance between the observer and the source is D_{OE}



C.1	Although the angle of deflection does not depend on parameter p, an Earth-based observer will be able to see more than one image only if the value of p is within a certain range. Find a condition on the value of the parameter p in terms of D_{ES}, D_{OE} , and temperature T , for an Earth-based observer to see more than one image of the object O	6.0pt

C.2	In case the observer sees more than one image, what is the angular separation between each pair? Find an expression in terms of D_{ES}, D_{OE} and $\delta\phi$	6.0pt

C.3	If $D_{OE} = 2D_{ES}$, determine the minimum size of an optical telescope needed to resolve this lensing event produced by GUT string.	4.0pt