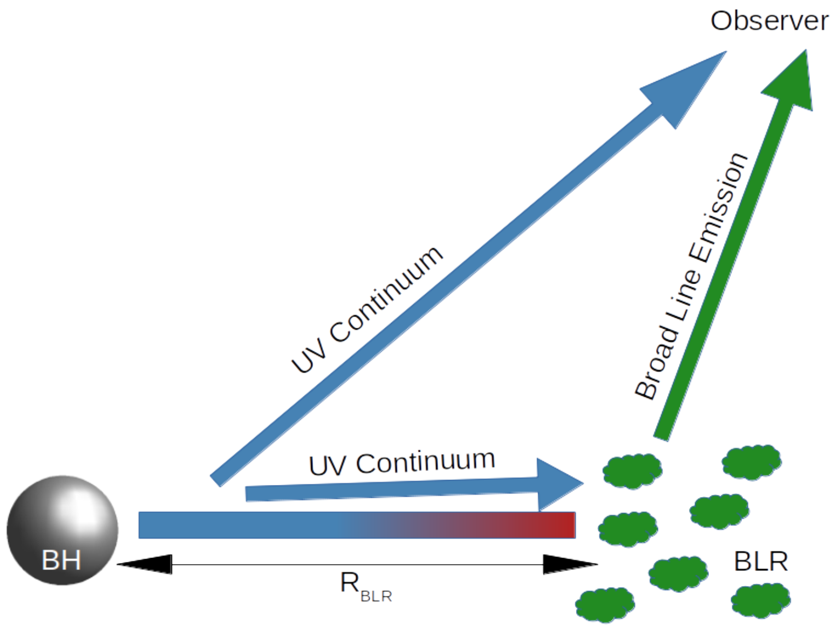
GeCAA - Data Analysis Solutions

September 21, 2020

1 AGN

It is believed that the accretion disk around supermassive black holes (BH) at galactic centres gives rise to UV thermal emission. This emission is associated with Active Galactic Nuclei (AGNs).

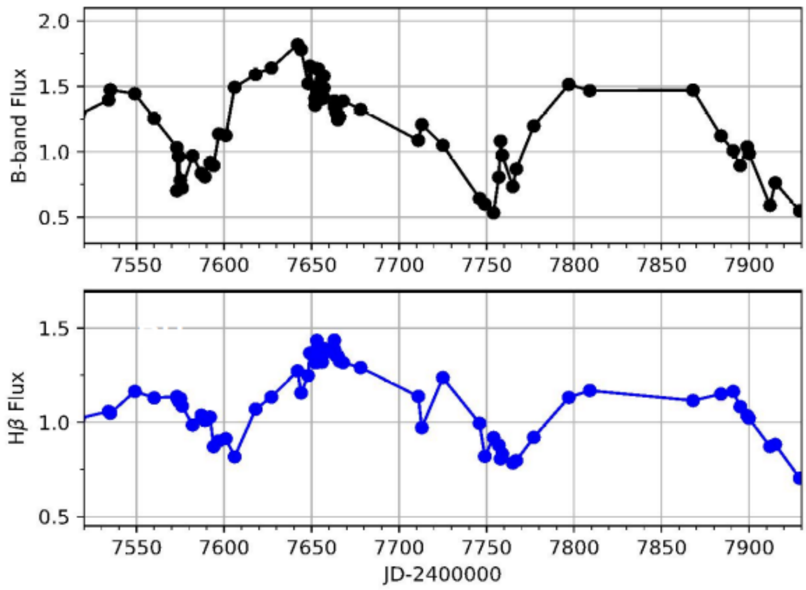
The optical spectra of bright AGNs show additional bright broad emission lines. Those emission lines arise from the dense gas in the Broad Line Region (BLR), which is ionized by the UV photons from the accretion disk. See the sketch to visualise this model.



We can assume that the flux of broad emission lines varies in response to the variation of the UV continuum with a time delay. This time delay should be proportional to the separation $R_{\rm BLR}$ between the BH and the BLR.

Assume that the size of the accretion disk is negligible as compared to $R_{\rm BLR}$.

(a) **(1 point)** Estimate the time lag (days) between the B-band continuum and broad emission line (H-$_\beta$) using the light curves shown below. The x-axis is in reduced Julian Dates (JD).



(b) **(3 points)** Estimate $R_{\rm BLR}$ in parsecs (pc).

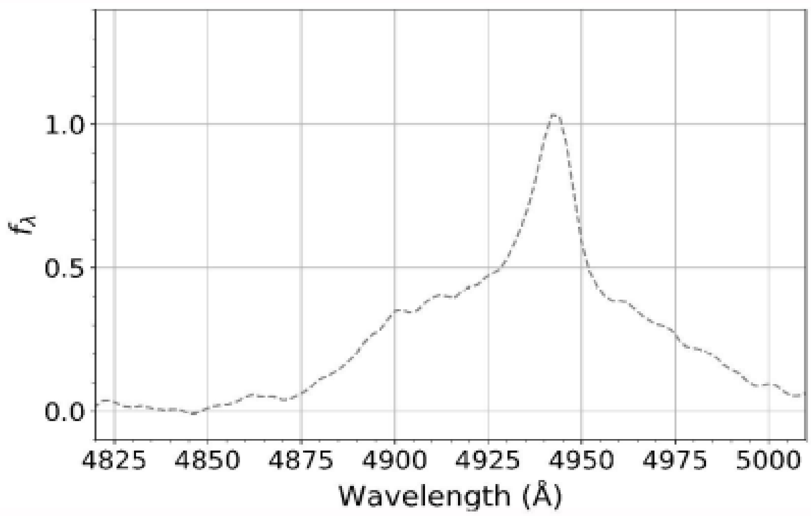
(c) **(2 points)** Estimate the angular separation of this region $\theta_{\rm BLR}$ (in arcsec) from the blackhole, if this AGN is 100 Mpc away from us.

It is possible to estimate the mass of the system using the Virial theorem, if the velocity dispersion of the gasses in the BLR and the size of the system are known. Assume that the masses of the accretion disk and broad line region are negligible, as compared to the black hole. The velocity dispersion v_σ may be estimated from the broadening of the given emission line. We will take the corresponding wavelength dispersion to be

$$\sigma = \frac{\rm FWHM}{2.35}$$

where FWHM is the full width at half maximum of the broad emission line.

(d) **(5 points)** Calculate the velocity dispersion v_σ in units of $\rm km\,s^{-1}$, from the spectral line shown below.



(e) **(4 points)** Calculate the mass of the central BH ($M_{\mathrm{vir,BH}}$) in a unit of $M_\odot$.

Solution

(a) Taking multiple reference points, we get that the BLR emission lags by about **20-25 days**. **1.0**

Answers from 15 to 25 days are acceptable.

(b) As the AGN is located very far, the time lag can be approximated as the time taken by UV emission to reach BLR region. Thus,

$$\begin{aligned} R_{\mathrm{BLR}} &= c\Delta t = 3 \times 10^8 \times 20 \times 86\,400 \\ &= 5.2 \times 10^{14}\,\mathrm{m} = (0.017 \pm 0.004)\,\mathrm{pc} \end{aligned}$$ **3.0**

(c) As this AGN is 100 Mpc away from us,

$$\begin{aligned} \theta_{\mathrm{BLR}} &= \frac{0.017}{100e6} \times 206265 \\ &= (3.5 \pm 0.9) \times 10^{-5}\,\mathrm{arcsec} \end{aligned}$$ **2.0**

(d) The FWHM is approximately (85 ± 5) Å and the peak is approximately at (4940 ± 5) Å **2.0**

$$v_\sigma = \frac{\sigma c}{\lambda_{\mathrm{peak}}} = \frac{\mathrm{FWHM} \times c}{2.35\lambda_{\mathrm{peak}}} = \frac{85 \times 3 \times 10^5}{2.35 \times 4940}$$ **2.0**

$$v_\sigma = (2200 \pm 140)\,\mathrm{km\,s^{-1}}$$ **1.0**

(e)

$$\begin{aligned} M_{\mathrm{vir,BH}} &= \frac{v_\sigma^2 R_{\mathrm{BLR}}}{G} = \frac{(2.2 \times 10^6)^2 \times 5.2 \times 10^{14}}{6.674 \times 10^{-11}} \\ &= 3.8 \times 10^{37}\,\mathrm{kg} \\ M_{\mathrm{vir,BH}} &= (1.9 \pm 0.7) \times 10^7 M_\odot \end{aligned}$$ **4.0**

2 Minor Planet

Table 1 gives ecliptic longitude (λ) and parallax (p) at different times (t), for a certain hypothetical minor planet. The baseline for the parallax is the diameter of the earth. The time is expressed in years and for your reference ecliptic longitudes of the Sun ($\lambda_\odot$) for the same dates are also given the table. Let us assume that the orbital inclination of this minor planet, with respect to the ecliptic, is negligible and the eccentricity of the Earth's orbit is negligible.

(a) **(38 points)** Calculate the coordinates of the minor planet in the heliocentric polar coordinate system and put them in a polar plot. The x-axis in the plot should be directed towards the initial position of the minor planet. Draw the major axis of the orbit of the minor planet.
Identify erroneous observation(s), if any.

(b) **(6 points)** Assuming the heliocentric orbit of the minor planet to be elliptical, determine

(i) the semimajor axis length a_p.

(ii) eccentricity e.

(iii) the period P.

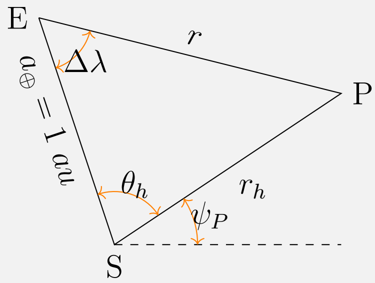
(c) **(6 points)** Estimate the errors in the values of P, a_p and e.

Table 1: Minor planet data

t [year]	λ [°]	$\lambda_\odot$ [°]	p [″]
2012.3	336.73	40.95	3.82
2012.6	3.44	134.83	7.24
2012.9	50.71	242.08	7.09
2013.4	94.52	64.84	2.40
2013.6	121.40	134.59	2.16
2013.9	154.31	241.82	2.75
2014.2	25.33	353.29	3.16
2014.5	148.51	99.04	1.99
2014.8	176.26	205.45	1.83
2015.0	216.33	280.19	2.03
2015.3	187.5	28.55	2.897

Solution

(a) We consider the $\triangle ESP$, the vertices of which are Earth (E), Sun (S) and the minor planet (P).



where $\Delta\lambda = |\lambda - \lambda_\odot|$. **2.0**

$$r \text{ (in m)} = \frac{R_\oplus}{p \text{ (in rad)}}$$
$$\therefore r \text{ (in au)} = \frac{206265 R_\oplus}{p \cdot a_\oplus}$$ **1.0**

By using the cosine rule and sine rule,

$$r_h^2 = a_\oplus^2 + r^2 - 2a_\oplus r\cos(\Delta\lambda)$$
$$r_h = \sqrt{a_\oplus^2 + r^2 - 2a_\oplus r\cos(\Delta\lambda)}$$ **1.0**
$$\frac{\sin\theta_h}{r} = \frac{\sin(\Delta\lambda)}{r_h}$$
$$\therefore \theta_h = \sin^{-1}\left(\frac{r\sin(\Delta\lambda)}{r_h}\right)$$ **1.0**

An angle ψ_P is defined as angle swept by the vector $\overrightarrow{SP}$ from an arbitrary x axis (see figure). Let us fix the direction of the x-axis as the direction $\overrightarrow{SP}$ at the initial moment $t = 2012.3$. In other words, $\psi_P(2012.3) = 0$. Using the same x-axis, let us define corresponding angle for the Earth, ψ_E. Thus, we have

$$\psi_P = \psi_E - \theta_h$$

If we take any two consecutive measurements,

$$\Delta\psi_E = \Delta\lambda_\odot$$ **2.0**

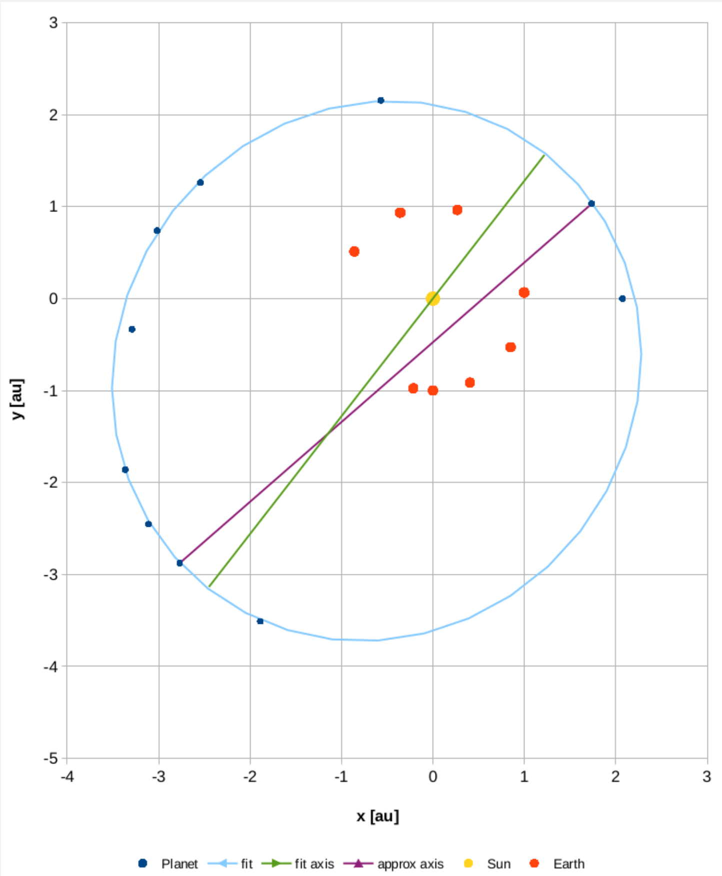
This is preferred over the conversion of time difference to angle as this data has better accuracy.
Using these formulae, one can find r, r_h, ψ_E and ψ_P for every measurement instance in the data. ψ_P and r_h can be used to plot locations of the minor planet.
Alternatively, one can also convert these to rectangular coordinates, i.e.

$$x = r_h\cos\psi_P,$$
$$y = r_h\sin\psi_P;$$

t [year]	**r** [au]	$\mathbf{r_h}$ [au]	$\sin\theta_\mathbf{h}$	$\theta_\mathbf{h}$ [°]	$\psi_\mathbf{E}$ [°]	$\psi_\mathbf{P}$ [°]	**x** [au]	**y** [au]
2012.3	2.302	2.073	-1.0000	270.00	270.00	0	2.073	0
2012.6	1.215	2.020	-0.4510	-26.81	3.88	30.69	1.737	1.031
2012.9	1.240	2.229	0.1097	6.30	111.13	104.83	-0.571	2.155
2013.4	3.664	2.839	0.6391	140.27	293.89	153.62	-2.543	1.262
2013.6	4.071	3.106	-0.2991	197.40	3.64	166.24	-3.017	0.739
2013.9	3.198	3.309	-0.9655	-74.92	110.87	185.79	-3.292	-0.334
2014.2	2.783	2.007	0.7357	132.63	222.34	89.71	0.010	2.007
2014.5	4.419	3.845	0.8735	119.13	328.09	208.96	-3.364	-1.862
2014.8	4.805	3.962	-0.5914	216.26	74.50	218.24	-3.112	-2.453
2015.0	4.332	3.994	-0.9738	-76.85	149.24	226.09	-2.770	-2.877
2015.3	3.035	3.985	0.2736	15.88	257.6	241.72	-1.888	-3.509

there are 11 rows. Two points for each row. **22.0**

Note. The parallax value for $p(2014.2) = 3.16''$ is obviously an outlier. Due to this, the corresponding position of the minor planet is also an outlier. Therefore, it should not be included in the plot or in the next analysis. **1.0**



4.0

These 4 points are for merely drawing the approximate polar or rectangular plot. Points for the line of apsides are separate (see below). Marking the earth positions or drawing the visually fit ellipse (blue line) is NOT expected.

One may try to identify a minimum and a maximum and maximum in the calculated values of r_h and take these as the aphelion and perihelion. In the figure these two points are shown and the line between drawn , i.e. the magenta line. Clearly these are not the aphelion and perihelion (as evidenced by the fact that the line connecting them does not pass through the focus), but they lie close. **1.0** **1.0**

We see that the points on either side of supposed aphelion are reasonably close (and the planet will be moving slower close to aphelion). However, there is a big gap between the second and third point, we take the aphelion position to be the correct one and draw the line of apsides to pass through aphelion and (0,0). (see below for the actual calculation of a_p that does not rely on the graphical identification of the line of the apsides.) **2.0**

(b) (i) Assuming that the points above are close to the perihelion and aphelion, we should realise that the distance between them is roughly equal to the length of the major axis. Using the cosine rule,

$$\begin{aligned}
\psi_1 &= 30.69^\circ \\
r_1 &= 2.020\,\text{au} \\
\psi_2 &= 226.09^\circ \\
r_2 &= 3.994\,\text{au} \\
\therefore \Delta\psi_P &= 226.09^\circ - 30.69^\circ \\
&= 195.40^\circ \\
2a_P &\approx \sqrt{r_1^2 + r_2^2 - 2r_1 r_2 \cos\Delta\psi_P} \\
&= \sqrt{2.020^2 + 3.994^2 - 2 \times 2.02 \times 3.994 \times \cos 195.40^\circ} \\
&= 5.965\,\text{au} \\
\therefore a_P &\approx 2.98\,\text{au}
\end{aligned}$$

3.0

(ii)

$$e = \frac{r_a - r_p}{r_a + r_p} \approx \frac{3.994 - 2.02}{3.994 + 2.02} \approx \frac{1}{3} = 0.33$$

2.0

Some students may visually fit an ellipse to the data and draw this line up to this visually fit ellipse. For the fit ellipse in the figure, the length of the major axis (green line) is $2a = 2.98$ au and $e = \frac{1}{3}$, rotated by 52°.

(iii) From Kepler's Laws,

$$\begin{aligned}
P &= \sqrt{a_P^3} = \sqrt{2.98^3} \\
&= 5.15\,\text{yr}
\end{aligned}$$

1.0

The semi-period will be 2.58 yr. Note that the time difference between our chosen points is only 2.4 yr. This is fine, as we can see from the best fit ellipse that these points are not exactly along the line of apsides.
At the same time, this is precisely the reason why one should not take Δt between these two points as the semi-period.

(c) The error in the semimajor axis is determined by the error in r_1 and r_2 which, in turn, depend on p and $\Delta\psi$. The latter is dominated by the uncertainty in the orientation of the ellipse (i.e., the assumption that r_1 and r_2 are close to the line of the apsides).

In order to take this into account we take as $\Delta\psi_P$ the difference with the value that corresponds to the difference in angles of the actual perihelion and aphelion (180°); i.e., $\delta\Delta\psi_P \approx 15/180$, corresponding to $\Delta(\Delta\psi) \approx 0.08\pi \approx 0.26$. The geocentric distance is determined via parallax, the other quantities are constants, so it may be written $r_g \propto p^{-1}$. The immediate consequence is $\delta r_g = \delta p$ where δ is the designation for the relative error. Thus,

$$\Delta r = r\delta r = r\delta p$$

This gives $\Delta r_1 = 1.01 \times 10^{-2}$ au and $\Delta r_2 = 1.997 \times 10^{-2}$ au.
The error in a_p is then

$$\Delta a_P = \frac{\sqrt{(r_1 - r_2\cos\Delta\Psi_p)^2\Delta r_1^2 + (r_2 - r_1\cos\Delta\Psi_p)^2\Delta r_2^2 + (r_1 r_2 sin\Delta\Psi_p)^2\Delta(\Delta\Psi_p)^2}}{2\sqrt{2}a_P}$$
$$= 0.04\,\text{au}$$
$$\therefore a_P = (2.98 \pm 0.04)\,\text{yr}$$ **3.0**

The uncertainty in the period can then be obtained from Kepler's law and is

$$\delta P = 3/2\delta a_p = 2 \times 10^{-2}$$
$$\therefore P = (5.2 \pm 0.1)\,\text{yr}$$ **1.0**

The eccentricity can be calculated from $r_p = a_p(1-e)$. Error propagation leads to

$$\Delta e = \sqrt{((1-e)\delta r)^2 + \left(\frac{1-e}{a_p}\Delta a_p\right)^2}$$
$$= 9 \times 10^{-3} \approx 0.01$$
$$\therefore e = 0.33 \pm 0.01$$ **2.0**

3 Hypervelocity stars

In recent years, a new field of research has emerged, that of Hypervelocity Stars (HVS for short). These are stars in our Galaxy (mostly at its outskirts), which are moving with excessive velocities and may be escaping from the Milky Way.

In this question, you will use spectroscopic and astrometric measurements in order to calculate the velocity of one such star, called "HVS1", consider its origin and whether it may escape the Galaxy.

Figure 1 shows a spectrum of HVS1 in the blue to UV part of the spectrum:

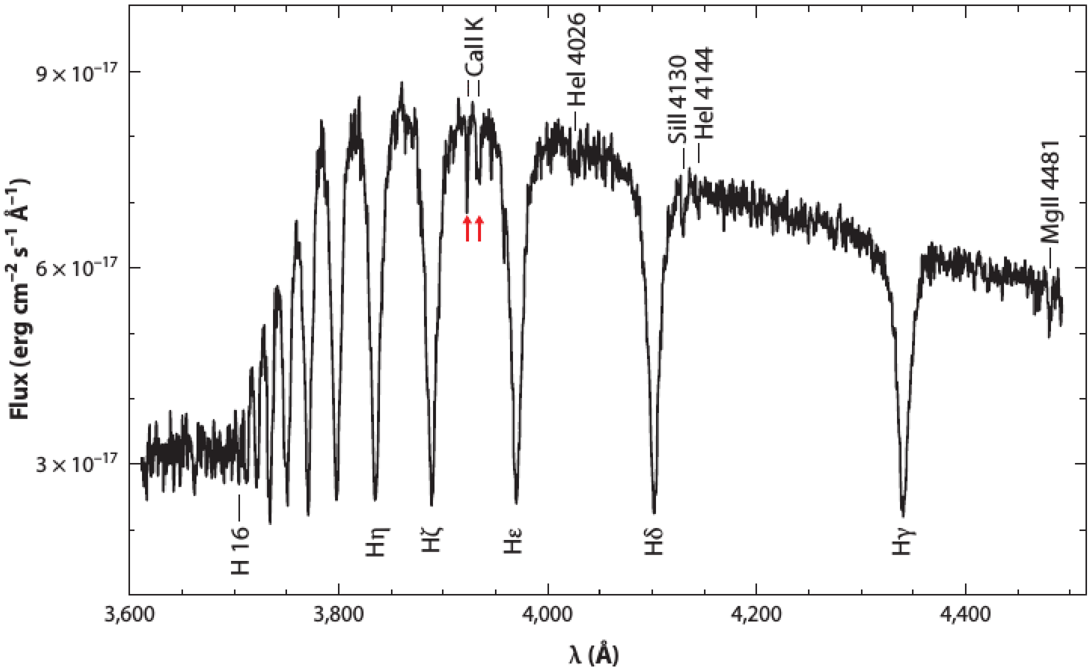


Figure 1: *The spectrum of HVS1 shifted to the rest frame of the star (i.e., there is no Doppler shift due to the motion of the star along the line of sight).*

(a) **(7 points)** Determine the spectral type of the star using the standard spectra in Appendix 3 and the absorption lines identified on the spectrum of HVS1. (Note that the spectrum above contains both stellar and interstellar absorption lines.)

Solution

By comparing with spectral charts in appendix 3, we notice that

- H-lines are the most promienent lines,
- He I feature at 4144 Å is bearly visible and
- Ca I feature around 4230 Å is not seen.

Thus, it may be reasonable to restrict the range of spectral types to B5-A5. **2.0**

We find that the ratio of the intensity of a pair of lines and compare it with the standard spectra. We see, Ca II/H$\epsilon \sim \frac{1}{4}$ **2.0**

For B8 standard spectra it is $\sim \frac{1}{5}$, for A0 standard spectra it is $\sim \frac{1}{3}$. **2.0**

Thus, we deduce that the spectral type lies somewhere between B8 and A0. One may write any spectral type in the range, say B9. **1.0**

(b) **(18 points)** Detailed modeling of the spectral lines places the star between luminosity classes V (Main Sequence) and IV (subgiant).

i. The apparent magnitude of the star in the visual band is $m_V = 19.84$. Find the absolute magnitude M_V of the star using Appendix 1 for the two possible luminosity classes.
You may ignore the uncertainty in m_V since the uncertainty in your calculation will be dominated by the uncertainty in M_V.

Solution
Appendix 1 gives absolute magnitude for stars of different spectral and luminosity classes. The relevant rows are A0 and B5. **1.0**
In order to estimate the absolute magnitude of a B9 star, interpolate between B5 and A0.

$$\begin{aligned} &\text{For MS:} \\ M(\mathrm{B9}) &= M(\mathrm{A0}) - \frac{(M(\mathrm{A0}) - M(\mathrm{B5}))}{5} \times (10-9) \\ &= 0.7 - \frac{(0.7 - (-1.1))}{5} = 0.7 - 0.36 \\ &= 0.34 \end{aligned}$$
2.0

$$\begin{aligned} &\text{For subgiants:} \\ M(\mathrm{B9}) &= M(\mathrm{A0}) - \frac{(M(\mathrm{A0}) - M(\mathrm{B5}))}{5} \times (10-9) \\ &= 0.1 - \frac{(0.1 - (-1.8))}{5} = 0.1 - 0.38 \\ &= -0.28 \end{aligned}$$
2.0

ii. For both these possible luminosity classes, calculate the star's distance from the Sun, ignoring interstellar absorption.

Solution

$$\begin{aligned} m_v &= M_v + 5\log(d) - 5 \\ \therefore d &= 10^{\frac{m_v - M_v + 5}{5}} \end{aligned}$$
1.0

$$\begin{aligned} &\text{For MS:} \\ d &\approx 79.4\,\mathrm{kpc} \end{aligned}$$
1.0

$$\begin{aligned} &\text{For subgiants:} \\ d &\approx 105.7\,\mathrm{kpc} \end{aligned}$$
1.0

iii. The galactic coordinates of HVS1 are $l = 227.335\,372\,67°$, $b = 31.331\,993\,86°$. Is the assunption of ignoring the interstellar absorption justified? Write "YES" or

“NO”.

Solution
Yes. HSV1 is off the galactic plane, well into the halo. interstellar absorption should not be very high. Thus, ignoring it will not affect the distance very much. **2.0**

iv. The Gaia mission of the European Space Agency has been mapping the Milky Way since 2014, measuring the parallax and proper motion of 1.5 billion stars to an accuracy between 0.04 and 0.1 milliarcseconds (mas). Could Gaia have measured the parallax of HVS1,

(A) if it is a MS star? Write “YES” or “NO”.

(B) if it is a subgiant? Write “YES” or “NO”.

Solution

(A) if it is a MS star,

$$\pi = 1/d \sim 0.01\text{mas} < \pi_{\text{GAIA}}$$

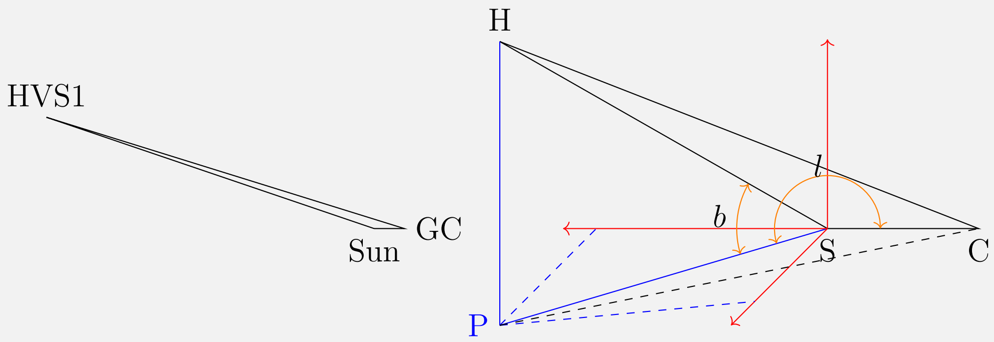
Hence, the answer is “**No**”. **1.0**

(B) If it is a subgiant, then the distance is even larger. Hence, the answer is again “**No**”. **1.0**

For the rest of this question, adopt the larger of the two distances you have calculated above.

v. Assume that the distance of the Sun from the Galactic center is 8.0 kpc. Make a rough sketch of the relative positions of HVS1, the Sun and the Galactic center. Use it to calculate the distance (r) of HVS1 from the Galactic center.

Solution



The left panel is to the scale. The right panel shows details of the drawing. **2.0**

$$
\begin{aligned}
d(HC)^2 &= d(HP)^2 + d(PC)^2 \\
&= d(HP)^2 + \left[d(SC)^2 + d(PS)^2 - 2 \cdot d(SC) \cdot d(PS)\cos(\measuredangle PSC)\right] \\
r^2 &= (d\sin b)^2 + r_\odot^2 + (d\cos b)^2 - 2r_\odot d\cos b\cos(360^\circ - l) \\
\therefore r &= \sqrt{d^2 + r_\odot^2 - 2r_\odot d\cos b\cos l} \\
&= \sqrt{105.7^2 + 8^2 - 2\times 8\times 105.7\cos(31.332^\circ)\cos(227.335^\circ)} \\
\therefore r &\approx 110\,\mathrm{kpc}
\end{aligned}
$$
4.0

(c) **(17 points)** Here, you will calculate the actual velocity of HVS1.

i. The spectrum in Fig1 shows two absorption lines due to Ca II. One is caused by the atmosphere of the star and the other is due to the interstellar medium. The shift of this line is due to the motion of the star with respect to the interstellar medium. Measure this Doppler shift and calculate the radial velocity of HVS1 with respect to the Sun.

Solution From the spectrum, we can estimate the line shift as

$$\Delta\lambda = (11 \pm 2)\,\text{Å},\ \ \lambda_0 = 3934\,\text{Å}$$
3.0

Doppler shift gives the heliocentric velocity

$$v_{\mathrm{rHC}} = \frac{c\Delta\lambda}{\lambda_0} = (840 \pm 150)\,\mathrm{km\,s^{-1}}$$
2.0

ii. We are interested in the velocity with respect to the Galactic center. For this we first need to take into account the velocity of the Sun due to the rotation of the Galaxy. The following equation transforms the velocity of a star of heliocentric radial velocity v_{rHC} to one in the Galactic rest frame (rf), v_{rf}:

$$v_{\mathrm{rf}} = v_{\mathrm{rHC}} + 11.1\cos l\cos b + 247.24\sin l\ cosb + 7.25\sin b,$$

where the speeds are measured in $\mathrm{km\,s^{-1}}$.
Find v_{rf} for HVS1.

Solution

$$
\begin{aligned}
v_{\mathrm{rf}} &= v_{\mathrm{rHC}} + 11.1\cos l\cos b + 247.24\sin l\ cosb + 7.25\sin b \\
&= 840 + 11.1\cos(227.335^\circ)\cos(31.332^\circ) \\
&\quad + 247.24\sin(227.335^\circ)\cos(31.332^\circ) + 7.25\sin(31.332^\circ) \\
v_{\mathrm{rf}} &= (680 \pm 130)\,\mathrm{km\,s^{-1}}
\end{aligned}
$$
2.0

iii. HVS1's proper motion has been measured as:

$$(\mu_\alpha, \mu_\delta) = (+0.08 \pm 0.26, -0.12 \pm 0.22)\,\mathrm{mas/yr}.$$

Calculate the tangential velocity component (in $\mathrm{km\,s^{-1}}$) of HVS1. (You may ignore the correction for declination as the star is near the celestial equator).

Solution
The length of an arc does not depend on the coordinate system. Thus,

$$\begin{aligned}
\Delta\mu &= \sqrt{\mu_\alpha^2 + \mu_\delta^2} \\
&= \sqrt{0.08^2 + 0.12^2} \\
\Delta\mu &= (0.144 \pm 0.340)\,\mathrm{mas/yr} && \mathbf{2.0} \\
v_\theta = d\Delta\mu &= \frac{105.7\times10^3\times3.086\times10^{13}\times0.144\times10^{-3}}{206\,265\times3.16\times10^7} \\
&= (72\pm170)\,\mathrm{km\,s^{-1}} && \mathbf{2.0}
\end{aligned}$$

iv. Calculate the velocity v_T of the star with respect to the Galactic center (magnitude in $\mathrm{km\,s^{-1}}$ and angle with respect to direction of the Galactic center).

Solution
The direction to the Galactic center is *also* perpendicular to the plane of the sky. Thus,

$$\begin{aligned}
v_\mathrm{T} &= \sqrt{v_\mathrm{rf}^2 + v_\theta^2} = \sqrt{680^2+72^2} \\
v_\mathrm{T} &= (684\pm210)\,\mathrm{km\,s^{-1}} && \mathbf{2.0}
\end{aligned}$$

i.e. it is almost radial.

$$\theta = \tan^{-1}\left(\frac{v_\theta}{v_\mathrm{rf}}\right) \approx \tan^{-1}(0.105) \approx 6^\circ \qquad \mathbf{2.0}$$

v. Assuming this star was born within the Galactic disc, use your calculation of the velocity to estimate where in the Galactic disc it is more likely to have come from:

(A) near to the Galactic center

(B) further out in the Galactic disc

Solution As the velocity is almost radial, extrapolating backwards, the star would be in galactic disk somewhere close to the galactic centre. Thus, the answer is **A**. **2.0**

(d) **(6 points)** From the energy considerations,

i. Write expression for the escape speed v_esc as a function of the distance from the Galactic center and the enclosed mass.

Solution

$$v_{esc} = \sqrt{\frac{2GM}{r}}$$

1.0

ii. Calculate the mass of the galaxy (in solar masses) within the radius of the distance of HVS1.

$$M_r = 4\pi\rho_0 r_c^2 \left[r - r_c \tan^{-1}\left(\frac{r}{r_c}\right)\right]$$

where $r_c \approx 8\,\mathrm{pc}$ is an constant of the equation and $\rho_0 = 1.396 \times 10^4 M_\odot \mathrm{pc}^{-3}$.

Solution

$$\begin{aligned} M_r &= 4\pi\rho_0 r_c^2 \left[r - r_c \tan^{-1}\left(\frac{r}{r_c}\right)\right] \\ &= 4\pi \times 1.396 \times 10^4 \times 8^2 \left[110 - 8 \times \tan^{-1}\left(\frac{110}{8}\right)\right] \end{aligned}$$

$M_r(110\,\mathrm{kpc}) \approx 1.2 \times 10^{12} M_\odot$

2.0

iii. Calculate the magnitude of the escape velocity at the distance of HVS1.

Solution

$$v_{esc} = \sqrt{\frac{2GM}{r}} = \sqrt{\frac{2 \times 6.674 \times 10^{-11} \times 1.2 \times 10^{12} \times 1.998 \times 10^{30}}{110 \times 10^3 \times 3.086 \times 10^{16}}}$$

$$= 311\,\mathrm{km\,s^{-1}}$$

2.0

iv. Is this a runaway star? Write "YES" or "NO".

Solution
$v_{\mathrm{rf}} > v_{esc} \implies$ **Yes**.

1.0

(e) **(2 points)** How long has it taken for HVS1 to reach this position?

Solution If HSV1 originated close to the galactic center and assuming is was ejected at the measured speed, it has been traveling for

$$t_{\mathrm{trav}} = \frac{r}{v_{\mathrm{rf}}} = \frac{110 \times 10^3 \times 3.086 \times 10^{16}}{680 \times 10^3} = 1.6 \times 10^8\,\mathrm{yr}$$

2.0

(f) **(3 points)** On the basis of the spectral type and the luminosity class of this star, estimate the age of HVS1 and compare this with your result in the previous part. Which one of the following statements about the origin of the star is true:

(A) The star was ejected when or shortly after it was formed.

(B) The star was ejected mid-way through its time on the Main Sequence.

(C) The star was ejected towards the end of its time on the Main Sequence.

Solution
Main Sequence life time for the star will approximately be

$$t \sim \frac{M}{L} \sim L^{-5/7}$$

Comparing with absolute magnitude of the Sun,

$$t \sim 10^8 \, \text{yr}$$

This is very similar to the time taken by the star to travel from the disk to present position. Thus, the anwer is **A**. **3.0**

(g) **(2 points)** Astronomers looking for HVS-s start by finding a sample of stars in the Galactic halo which are of a spectral type similar to that of HVS1. Explain why by choosing which one of the following statements is true:

(A) Stars of this spectral type are young and so belong to the native population of the halo.

(B) Stars of this spectral type are old and so belong to the native population of the halo.

(C) Stars of this spectral type are young and so **do not** belong to the native population of the halo.

(D) Stars of this spectral type are old and so **do not** belong to the native population of the halo.

Solution Stars of early type are very young, so they cannot belong to the native population of the halo. They must be coming from elsewhere. As a result, they may be moving very fast and some of them may be on their way to abandoning the Milky Way.
$\Longrightarrow$ C. **2.0**